VOLUME XI NOVEMBER, 1915 NOS. 1 AND 2

BIOMETRIKA

ON THE ASSOCIATION OF TEMPERATURE, PULSE AND RESPIRATION WITH PHYSIQUE AND INTELLIGENCE IN CRIMINALS: A STUDY IN CRIMINAL ANTHROPOMETRY

BY MADELINE H. WHITING

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(1) *Introductory.* In a recent investigation as to school children by Williams, Bell and Pearson*, it was shown that in a series of seven schools temperature was negatively correlated with weight for constant age, and further in six out of the seven school series negatively correlated with stature for constant age. The correlations were on the whole small, but definitely significant. In the great public schools the temperature appeared, allowing for age, to be lower than in the elementary schools. An association was thus suggested between intelligence and temperature, the more intelligent having the lower temperature. This result might well be considered as spurious, and due to differences of nurture, particularly to differences of nutrition. On the other hand various writers have asserted that *low* temperatures are associated with *low* intelligence, and it is usually stated

* "A Statistical Study of Oral Temperatures in School Children," *Drapers' Company Research Memoirs. Studies in National Deterioration*, No. IV (Cambridge University Press). See pp. 57 and 59.

that the temperature of idiots is lower than that of normals. Such a conclusion might easily be reached if great care were not taken in keeping the mouths of the mentally defective closed. Norsworthy has given a descending scale of temperature from brightest girls to mediocre girls and so through mental defectives to idiots*. But the numbers dealt with were very inadequate, and the time of insertion very short. It appeared therefore well worth while considering further certain temperature problems, and the Biometric Laboratory was lucky enough to have placed at its disposal observations by Dr Charles Goring on 500 convicts. Of these convicts 400 were of normal intelligence and 100 were classed as weak-minded. In the bulk of cases the temperatures were taken twice at an interval of 14 days and upwards, the temperature of the room was also taken. Further the pulse and respiration were observed on both occasions. The examinations were all made at the same time, about a quarter of an hour after early dinner, and in the same condition,—the prisoner would be sitting in his cell, either reading, writing or doing nothing. When the thermometer had been inserted under his tongue, the medical officer would count the frequency of his pulse during one minute, and then would count respirations during one minute by observation of abdominal movement; the prisoner himself had of course no knowledge that this observation was being made. Besides the above data, the age, stature, weight, nature of the crime and previous convictions, the existence of special disease, the general health, the physical condition with regard to nutrition and muscularity, as well as the class of prison labour were all recorded. Only some of these data are discussed in the present paper.

Of the characters dealt with, stature, age, weight, pulse, respiration and temperature are quantitatively determined, intelligence is in two classes, normal and feeble-minded; the other data are in broad categories. Thus General Health is classed as "Poor," "Indifferent," or "Good"; for physical condition there are two divisions, each in alternate categories, (i) "Fat" or "Thin" and (ii) "Muscular" or "Weak"; labour done by the convict is classed into "Heavy" and "Light"; and this forms an adequate classification by which to test health conditions and other physical categories.

(2) *Temperature.* The temperatures were oral, taken with a Kew certified minute thermometer inserted under tongue with closed mouth for three minutes. None of the subjects were hospital invalids, all at the time of the observations were getting about, but of course a good many were suffering from chronic disease or disorders of one kind or another. The following classification of these diseases and disorders was made:

I. Syphilis: disease apparently inactive at time of observation.

II. Chronic Bronchitis or Asthma, or both: free from attack at time of observation.

* "The Psychology of Mentally Defective Children," *Archives of Psychology*, November, 1906. See also Pearson: *Mendelism and Mental Defect*, No. III (Cambridge University Press), p. 27.

III. Organic Heart Disease.

IV. Chronic Tubercular Disease: quiescent at time of observation.

V. Cardio-vascular degenerative changes.

VI. Epilepsy.

VII. Functional Heart Disorders without organic disease, and described as "weak heart."

VIII. Kidney degenerative changes: indicated by albuminuria, etc.

IX. Other pathological states: Necrosis of bone of old standing; diabetes, active at time of observation; tabes dorsalis (active); general paralysis of the insane (active); chronic rheumatism (active); urethral stricture, resulting from gonorrhoea; previously insane; goître (active); cirrhosis of liver (active); hemiplegia, resulting from a stroke.

927 temperatures were taken on 500 individuals, 427 being repeated temperatures, the intervals between observations varying from 2–3 weeks to 2–3 months. 326 temperatures were those of criminals suffering from the diseases and disorders classed as above, the remaining 601 temperatures may be taken as those of fairly normal individuals, differentiated only from the same class of the population by the fact of their criminality and by the action of prison diet, discipline and labour. Out of the 400 mentally normal convicts 34·5 % were suffering from one or another disorder, and of the 100 weak-minded convicts, 27 %. This is explained by the fact that Parkhurst, where the observations were made, is largely a prison to which the physically or mentally feeble are sent, and accordingly it is more than probable that the weak-minded in general suffer at least as much from disorder and disease as the normal-minded criminal. Taking apparently inactive syphilis alone we find 12·75 % of the normal-minded and 11·00 % of the weak-minded* suffering from non-active disease, a nearer ratio of equality. It would be of great value if a Wassermann test could be made throughout a large convict establishment. It seems probable that at least 10 % of the inmates would be found to be syphilitic. Making due allowance for the class from which the bulk of criminals are drawn, this does not suggest that much more than 5 %–6 % of the non-criminal population are likely to be affected, and compares on the whole favourably with the high estimates published of the syphilised population in Germany†. The following table sets forth our results for mean temperatures:

* One case of general paralysis of the insane included, and in both classes the probably syphilitic also counted.

† Fritz Lenz: "Ueber die Verbreitung der Lues...," *Archiv für Rassen- und Gesellschafts-biologie*, Bd. VII. S. 306–327, 1910.

TABLE A. *Mean Temperatures of Various Classes.*

Class	No. of Temperatures	Mean Temperatures
"Weak Heart" (VII)	20	98·585 ± ·068
Weak-minded	191	98·547 ± ·022
Epilepsy (VI)	27	98·537 ± ·059
Cardio-vascular changes (V)	36	98·408 ± ·050
Normally Healthy Criminals	601	98·383 ± ·013
Whole Population	927	98·373 ± ·011
Chronic Bronchitis, Asthma (II) ...	51	98·353 ± ·043
Kidney changes (VIII)	14	98·350 ± ·082
Syphilis (I)	97	98·342 ± ·031
Organic Heart Disease (III)	32	98·278 ± ·054
Chronic Tubercular Disease (IV) ...	22	98·241 ± ·065
Other pathological conditions (IX) ...	27	98·170 ± ·059

Now it must be at once admitted that many of the series here are so small, that we cannot say that the temperature differences, still less the general temperature order, are significant, but certain differences do appear significant. For example, the weak-minded do appear to have a significantly *higher* average temperature than normally healthy criminals. The difference,

$$\text{weak-minded} - \text{normally healthy} = \cdot 164 \pm \cdot 026,$$

is clearly significant numerically, but this does not show that the higher temperature is inherent in the weak-mindedness. We have already seen that of the weak-minded 27 % were suffering from one form or another of disorder, while we are comparing them against a normally healthy criminal population. The only groups, however, that could tend to *raise* the temperature of the weak-minded are V, VI and VII. To these groups the weak-minded contribute

V.	Cardio-vascular degenerative changes:	4 temperatures.
VI.	Epilepsy:	16 ,,
VII.	"Weak Heart":	0 ,,

These 20 temperatures have therefore been removed and we find:

$$\text{Mean temperature of weak-minded} = 98 \cdot 541 \pm \cdot 021.$$

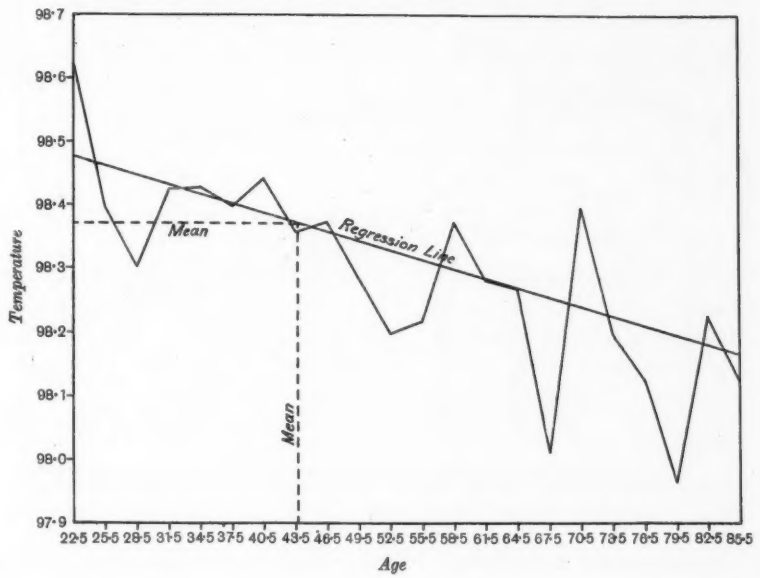
Thus the removal of the epileptic has only lowered very slightly the temperature of the weak-minded group. That has indeed 8·4 % of the temperatures of the epileptic as against only 1·2 % in the group of normal-minded criminals*, but the difference of temperature is only in the very smallest degree due to this excess of epilepsy. One point, however, of some apparent importance had it been based on more numerous data was observed, namely, that the 18 temperatures of the weak-minded who were suffering from syphilis, apparently inactive at the time of observation, gave a high mean, i.e. 98·683, as against the fact that the general

* Reckoned on individuals, not on temperatures observed, the percentages are 9·0 % and 1·5 % respectively.

syphilised population had if anything a rather subnormal temperature; but the removal of these 18 temperatures gave for the weak-minded population a mean-temperature of 98·533 ± ·021, which although slightly reduced is still significantly greater than that of the normal criminal population. I think we must conclude that none of the observed pathological states is the source of the higher temperature of the weak-minded, but whatever that source is, it is intensified in the presence of either epilepsy or syphilis*.

In the paper by Williams, Bell and Pearson, already cited, it is shown that body temperature falls with age up to the adult stage. From the present data we see that from age 22·5 to age 85·5 the fall is continued. Table XXXIV gives

DIAGRAM I. Regression Line of Temperature on Age for Criminals.



the data and Diagram I shows the plotted regression line. If T be the temperature in degrees Fahrenheit, A the age in years, the equation to the regression line is:

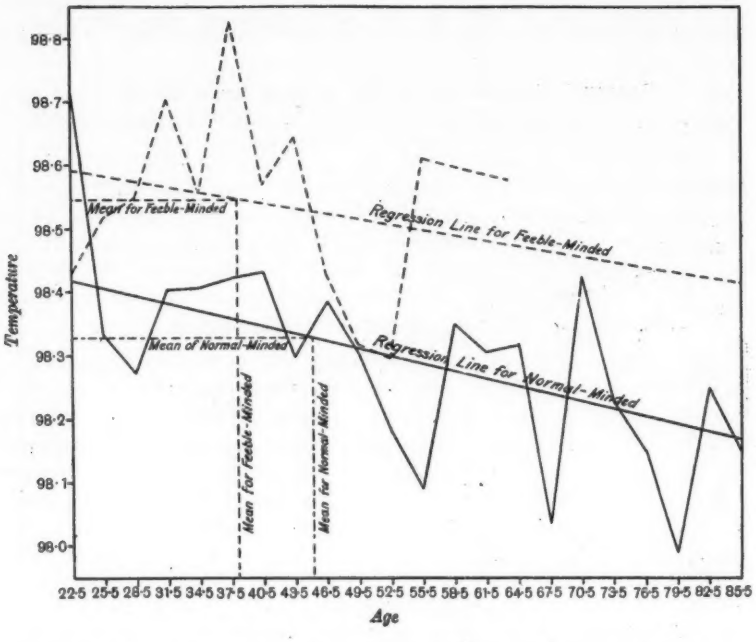
$$T = 98{\cdot}5915 - {\cdot}004989\,A,$$

which indicates that the temperature falls ·005 or $\frac{1}{200}$ of a degree per year. The problem then arises—can the difference of temperature between the weak-minded

* Weak-minded epileptics, 98·669; criminal epileptics in general, 98·537; weak-minded syphilitics, 98·683; criminal syphilitics in general, 98·342. These results are open to the interpretation that the weak-mindedness may be owing to cerebral injury due to the disease, and that cerebral disorders are more apt than other forms of these diseases to be accompanied by higher temperatures.

and normal criminal be solely an age difference? An examination of Table B shows that the mean age of the weak-minded is 38·16 years and of the normal-minded 44·86 years, or the former are younger by 6·70 years. This would correspond only to an average greater temperature of ·034 degrees as against the observed difference of ·164 degrees. The temperature difference of weak- and normal-minded is therefore not due to age difference. This is confirmed in Diagram II, where

DIAGRAM II. Regression Lines of Temperature on Age for Feeble- and Normal-Minded.



the observations and the calculated regression lines for the weak-minded and the normal-minded groups* are plotted alongside each other. Notwithstanding the irregularities due to the smallness of the sample, it will be seen that at *every* age the mentally defective have a higher temperature than the normal-minded, and further that, as averaged out by the regression lines, the difference at each age is sensibly constant (the regression lines are practically parallel) and equal to 0°·2.

We conclude that weak-minded adults have a higher temperature than normal-minded adults and in this respect differ from mentally defective children who are

* The equation to the regression line of the weak-minded group is $T = 98{\cdot}6535 - {\cdot}002796A$, and of the normal-minded group $T = 98{\cdot}5063 - {\cdot}003939A$.

stated by Professor Norsworthy* to have a lower temperature than normal-minded children, or else Norsworthy's material by including idiots was differentiated from weak-minded criminals, or again was unreliable owing to the brief period of insertion of the thermometer, and possibly to insufficient care in keeping the mouths of the mentally defective children closed during the observation. It is not possible in our case to account for this temperature difference by either (i) difference of age† or (ii) difference of observed pathological states other than those summed up in the word weak-minded.

One of the noteworthy points of Table A is that when we pass from one or other type of heart trouble (V or VII) and from the epileptic and weak-minded groups to chronic and organic disorders and diseases of a degree of activity which does not involve hospital segregation, we find, without laying too great stress on the individual order, that they tend on the whole to subnormal temperatures.

(3) *Further Differentiated Characters of the Weak-minded and Normal-minded Populations.*

A study of Table B shows that the weak-minded criminal is differentiated from the normal-minded‡ in a number of other characters beyond body temperature.

TABLE B.

Table of Means, Standard Deviations and Coefficients of Correlation of the Chief Variates dealt with.

Variate	Unit	Mean		Standard Deviation		Coefficient of Variation	
		Whole Population	Weak-minded	Whole Population	Weak-minded	Whole Population	Weak-minded
Age	Years	43·518±·438	38·160±·717	14·510±·309	10·627±·507	33·34±·79	27·85±1·43
Temperature	Degs. Fahr.	98·373±·011	98·547±·022	·486±·008§	·454±·016	·49±·01	·46± ·02
Pulse	Per minute	74·215±·245	77·620±·578	11·062±·173	11·852±·409	14·89±·24	15·27± ·54
Respiration	Per minute	18·985±·075	19·679±·168	3·380±·053	3·433±·119	17·80±·29	17·45± ·62
Height	Inches	65·281±·082	64·788±·185	2·682±·058	2·706±·131	4·11±·09	4·18± ·20
Weight	Lbs.	142·147±·516	134·397±·951	16·996±·365	13·881±·672	11·96±·26	10·33± ·51

* *loc. cit.* on our p. 2.

† This is confirmed by the correlation −·229±[·030] for mentality and temperature for constant age. See Table C.

‡ The term "normal-minded" is throughout this paper used of that portion of the criminal population not classed as weak-minded, and is not intended to serve as the basis of any opinion as to whether the non-weak-minded criminal is or is not mentally differentiated from the general non criminal normal-minded population.

§ It will be noticed that the value of this standard deviation is in very close agreement with those found for first and second temperatures in the case of children by Williams, Bell and Pearson. See *loc. cit.* p. 10, where $\sigma_{t_1} = ·5343$ and $\sigma_{t_2} = ·4803$.

For if he significantly exceeds or falls short of, on the average, the *whole* population, he will *a fortiori* be differentiated from the normal-minded population*. We have the following results·

Difference of Means: (Weak-minded—whole population)		
	Temperature	·174 ± ·025.
	Pulse	3·205 ± ·628.
	Respiration	·694 ± ·184.
	Height	− ·493 ± ·202.
	Weight	− 7·750 ± 1·305.

We conclude from these results that while the weak-minded criminal probably but not very certainly has a somewhat less stature†, he has quite markedly less weight, and this is combined with a higher temperature, a quicker pulse and a quicker respiration.

We now come to precisely the same point as we reached in the problem of differential temperatures. We ask: How far are these differences explicable on the basis of the recognised difference in age between the weak-minded and the total population? From Table C (p. 11) we extract the following correlations and we give the corresponding regressions:

Variates	Correlations	Regression of Second Variate on Age		
		Per Year	Per 5·36 Years	Comparable Difference
Age and Temperature	− ·150 ± ·022	− ·005,026	− ·0269	·174
Age and Pulse	+ ·121 ± ·022	+ ·092,400	+ ·4951	3·205
Age and Respiration	+ ·077 ± ·022	+ ·017,887	+ ·0958	·694
Age and Height	+ ·023 ± ·030	+ ·004,177	+ ·0224	− ·493
Age and Weight	+ ·136 ± ·030	+ ·159,532	+ ·8548	− 7·750

* Let the means, total frequencies and standard deviations of the general, the normal-minded and the weak-minded population be $\bar{y}$, $\bar{n}$ and $\bar{w}$, f_y, f_n, f_w, σ_y, σ_n, and σ_w, and let β be the multiple (say, about 3) which a quantity must be of its probable error to be considered significant. Then in the usual way $\bar{n}-\bar{w}$ is significant if

$$\bar{y}-\bar{w} \text{ is} = \text{or} > \beta\,(\cdot 67449)\sqrt{\frac{\sigma_y^2}{f_y}+\frac{\sigma_w^2}{f_w}\left(1-\frac{2f_w}{f_y}\right)}\Big/\sqrt{1+\frac{(\cdot 67449\beta)^2 f_w}{f_y(f_y-f_w)}}.$$

Now with $\beta=3$ and our numbers, roughly $f_w=190$, $f_y=930$, we have

$$\sqrt{1+\frac{(\cdot 67449\beta)^2 f_w}{f_y(f_y-f_w)}}=1\cdot 0005,$$

and this expression may be therefore taken as unity. Hence if we compare

$$\bar{y}-\bar{w} \text{ with } \cdot 67449\sqrt{\frac{\sigma_y^2}{f_i}+\frac{\sigma_w^2}{f_w}},$$

and it is significant it will certainly be significant if compared with

$$\cdot 67449\sqrt{\frac{\sigma_y^2}{f_y}+\frac{\sigma_w^2}{f_w}\left(1-\frac{2f_w}{f_y}\right)},$$

which is a smaller quantity. See *Biometrika*, Vol. v. pp. 181–183.

† Worked out from the fuller formula the result is −·493 ± ·143, or the difference is greater than three times its probable error and accordingly may be considered probably significant.

Now it will be seen that while all these correlations are significant except age and height*, none of them are of any considerable magnitude, or capable of producing any appreciable influence on the observed comparable differences. Further since pulse and respiration become very slightly quicker with age, they tend to emphasise and not to reduce the observed differences between these characters in weak-minded and normal-minded. Age might account for one-ninth of the observed difference in weight, or roughly for less than 1 lb. out of the nearly 8 lbs. difference observed; and, as before, age accounts for a very inconsiderable portion of the observed temperature difference. We are thus justified in concluding that the observed physical and physiological differences are quite independent of age differences†. We are able to confirm these results by another method of investigation, which well illustrates the insignificance of the age correction. In our opinion "weak-mindedness" must depend largely on the personal equation of the recorder‡, and there is no definite bounding line between the weak- and the normal-minded. In other words we may consider mental state as an approximately continuous variate, and one not widely divergent from the Gaussian in frequency distribution. Now it is known that in school-children the feeble-minded do not amount to 2 %§; and it is probable that in the community at large their number does not even reach this. On the other hand the criminal class is largely recruited from the less-intelligent section of the general population. Among inebriate and prostitute women probably 40 % to 60 % may be classed as feeble-minded‖, and though among male criminals such a high percentage is not reached, they must bulk very considerably. Goring in his invaluable *Report*¶ considers the percentage to be 10 to 15. In our present data we have 100 weak-minded individuals and 400 normal-minded, or we have taken 20 % and calculated the correlations between intelligence and the physical and physiological characters on this basis. But it is of interest to observe what order of change would be made in our results by taking a lesser percentage. Consider, for example, Table XXXV for intelligence and temperature. Here the temperatures of the weak-minded

* Of course the correlation of age and height is largely misleading, the younger criminals are still growing, the older are slowly losing height. Thus the regression is really skew. For example, the value of the correlation ratio η for height on age is ·147. If, however, we take the correlation of age and height for criminals of over 26 years, at which age other investigations (see *Biometrika*, Vol. I. p. 47) show that height begins to diminish, the negative correlation is only −·01195 corresponding to merely a loss of ·0022 inches per year of past prime life, and therefore quite negligible.

† Except in the matter of age and weight Table B shows us that judged either by absolute or relative variability there is no practical difference in variability between weak-minded and normal-minded criminals.

‡ On the continuity of Weak-mindedness, see *Questions of the Day and the Fray*, No. VIII. "The Continuity of Mental Defect," by Karl Pearson and Gustav A. Jaederholm; No. IX. "On the Graduated Character of Mental Defect," by Karl Pearson (Cambridge University Press).

§ See *A Preliminary Study of Alcoholism in Adults*, by Amy Barrington and Karl Pearson (Cambridge University Press), p. 9.

‖ *A Second Study of Extreme Alcoholism in Adults*, by David Heron (Cambridge University Press), p. 15.

¶ See *The English Convict, a Statistical Study* (Wyman and Sons), p. 254.

form 20·6 % of the 927 recorded temperatures; we will raise the normal-minded temperatures to 1719 by multiplying that row by the factor 2·336 retaining, however, only unit frequencies. We find approximately:

Table XXXV, modified to give 10 % weak-minded only.

Temperature.

Intelligence	96·5—96·6	96·7—96·8	96·9—97·0	97·1—97·2	97·3—97·4	97·5—97·6	97·7—97·8	97·9—98·0	98·1—98·2	98·3—98·4	98·5—98·6	98·7—98·8	98·9—99·0	99·1—99·2	99·3—99·4	99·5—99·6	99·7—99·8	99·9—100·0	Totals
Weak-minded	—	—	—	—	1	4	10	12	24	25	34	31	24	15	8	2	1	—	191
Normal-minded	2	5	4	7	49	70	145	201	236	299	248	236	114	61	23	12	5	2	1719
Totals	2	5	4	7	50	74	155	213	260	324	282	267	138	76	31	14	6	2	1910

Here the correlation r (found by the bi-serial method) is for this 10 % of the weak-minded

$$r = -\cdot231 \pm \cdot030,$$

whereas with the 20 % value of the weak-minded

$$r = -\cdot258 \pm \cdot030.$$

We conclude from this result that even halving the percentage of the feeble-minded will only alter the correlation by less than its probable error, and that accordingly for the purpose we have in hand the bi-serial values of the correlations based on 20 % of weak-mindedness will give sufficiently accurate results.

The reader is now asked to examine the last six lines of Table C. He will observe that, while nowhere very important, all the correlations of mentality with age, temperature, pulse, respiration, height and weight are significant; that the most important is weight, and that the signs are in agreement with the conclusions already reached, i.e. those with less mentality, or the feeble-minded, have less weight, height and age*, but greater temperature and quicker pulse and respiration. Examining the second column, which gives for the same variates the partial correlations for constant age, we see that no very substantial changes are made in the crude values; they all remain still significant, and this could have been anticipated by merely noting the small age and variate total correlations at the top of the first column. We conclude again that the physical and physiological characters dealt with are significantly related to feeble-mindedness, but the relationships are of such a low degree that they would be of absolutely no service for the purpose of

* The extent to which this lesser age of the weak-minded criminal is due (*a*) to earlier average conviction, resulting from earlier crime or easier capture, (*b*) to more frequent transfer to other institutions, or (*c*) to earlier death is of course not determinable from the present data. But it has to be borne in mind that fraudulent offenders convicted late in life belong to a different social class, and are rarely feeble-minded. This class has usually greater stature and weight.

TABLE C. *Physiological, Physical and Mental Correlations.*

Variates Correlated	Total Correlations	Partial Correlations									
		Age Constant	Temperature Constant	Pulse Constant	Respiration Constant	Height Constant	Weight Constant	Age and Respiration Constant	Age and Weight Constant	Age and Height Constant	Weight and Height Constant
Age and Temperature	−·150 ±·022	—	—	−·195 ±·021	−·142 ±·030	−·151 ±·021	−·158 ±·021	—	—	—	−·160 ±·021
Age and Pulse	+·121 ±·022	—	+·174 ±·022	—	+·117 ±·022	+·124 ±·022	+·107 ±·022	—	—	—	+·097 ±·022
Age and Respiration	+·077 ±·022	—	—	—	—	+·081 ±·022	+·090 ±·022	—	—	—	+·083 ±·022
Age and Height	+·023 ±·030	—	—	—	—	—	−·070 ±·030	—	—	—	—
Age and Weight	+·136 ±·030	—	—	—	—	+·151 ±·030	—	—	—	—	—
Temperature and Pulse	+·288 ±·020	+·312 ±·020	—	—	+·283 ±·020	+·289 ±·020	+·286 ±·020	+·305 ±·020	+·308 ±·020	+·314 ±·020	+·286 ±·020
Temperature and Respiration	+·142 ±·022	+·156 ±·022	—	+·130 ±·022	—	+·144 ±·022	+·146 ±·022	—	+·163 ±·022	+·158 ±·022	+·144 ±·022
Temperature and Height	+·003 ±·022	+·007 ±·022	—	—	—	—	−·027 ±·022	—	−·039 ±·022	—	—
Temperature and Weight	+·043 ±·022	+·065 ±·022	—	—	—	+·051 ±·022	—	—	—	+·075 ±·022	—
Pulse and Respiration	+·060 ±·022	+·070 ±·022	+·020 ±·022	—	—	+·049 ±·022	+·071 ±·022	—	+·081 ±·022	+·059 ±·022	+·052 ±·022
Pulse and Height	−·078 ±·022	−·081 ±·022	—	—	—	—	−·178 ±·022	—	−·172 ±·022	—	—
Pulse and Weight	+·114 ±·022	+·099 ±·022	—	—	—	+·197 ±·021	—	—	—	+·181 ±·022	—
Respiration and Height	−·144 ±·022	−·146 ±·022	—	—	—	—	−·113 ±·022	—	−·108 ±·022	—	—
Respiration and Weight	−·089 ±·022	−·101 ±·022	—	—	—	+·007 ±·022	—	—	—	−·020 ±·022	—
Height and Weight	+·580 ±·020	+·583 ±·020	—	—	—	—	—	—	—	—	—
Mentality and Age	+·264 ±·041	—	—	—	—	+·263 ±[·041]	+·234 ±[·041]	—	—	—	+·230 ±[·041]
Mentality and Temperature	−·258 ±·020	−·229 ±[·030]	—	—	—	−·262 ±[·030]	−·288 ±[·030]	—	−·262 ±[·030]	−·233 ±[·030]	−·289 ±[·030]
Mentality and Pulse	−·223 ±·030	−·266 ±[·030]	—	—	—	−·214 ±[·030]	−·276 ±[·030]	—	−·313 ±[·029]	−·264 ±[·030]	−·289 ±[·030]
Mentality and Respiration	−·148 ±·031	−·175 ±[·031]	—	—	—	−·129 ±[·031]	−·127 ±[·031]	—	−·153 ±[·031]	−·160 ±[·031]	−·133 ±[·031]
Mentality and Height	+·154 ±·043	+·153 ±[·043]	—	—	—	—	−·044 ±[·044]	—	−·031 ±[·044]	—	—
Mentality and Weight	+·324 ±·040	+·304 ±[·040]	+·347 ±[·040]	+·361 ±[·040]	+·315 ±[·040]	+·291 ±[·041]	—	—	—	+·274 ±[·041]	—

The partial correlations were all calculated from values of the total correlations to four figures. The probable errors of the bi-serial correlations were calculated by Soper's formula (see *Biometrika*, Vol. x. p. 384). Those of the partial coefficients involving bi-serial values of total coefficients were calculated on the assumption that the partial coefficients had been calculated directly from bi-serial tables and are placed in squared brackets to indicate that they are only approximative.

diagnosis, or for discriminating the weak-minded from the normal-minded. An examination of the Tables XXXVII, XXXVIII and XLII, XLIII shows that the range of the normal in every case is practically coextensive with, or more extensive than, that of the feeble-minded. This is in good accordance with the results already reached by Jaederholm and Pearson from other data: see *Questions of the Day and Fray*, VIII. "The Continuity of Mental Defect," and IX. "On the Graduated Character of Mental Defect." With regard to the further partial correlations, it will be seen that the relationships between the physiological variates (temperature, pulse and respiration) and mentality are not much modified by making the physical variates constant. The relationship between mentality and temperature reaches its maximum for constant weight and height, but the gain is only about its probable error; that for mentality and pulse is more considerable being three times the probable error for constant age and weight; while that for mentality and respiration is a maximum for constant age and height, but the increase is less than half the probable error. Finally it may be noted that the relationship of mentality to weight is somewhat intensified if the temperature and pulse rates be made constant.

In no case, however, do we reach correlations of the slightest importance for diagnostic purposes.

(4) *On the Interphysiological Correlations.*

The next point to which we must turn our attention is the relationship between temperature, pulse and respiration. We have the following series of correlations from Table C.

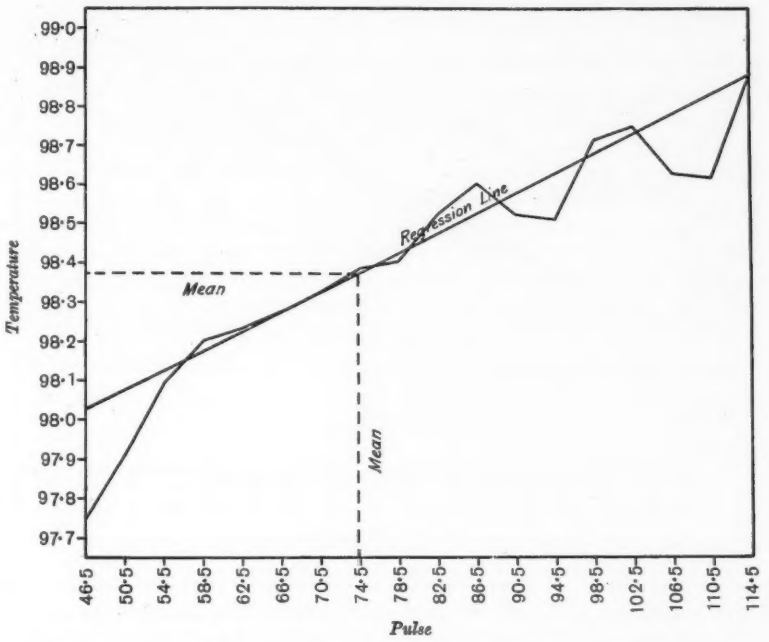
Correlated Pair	Crude Correlations	Partial Correlations for Constant Age and Weight	Partial Correlations for Constant Age and Height
Temperature (T) and Pulse (P)	$+\cdot288\pm\cdot020$	$+\cdot308\pm\cdot020$	$+\cdot314\pm\cdot020$
Temperature (T) and Respiration (R)	$+\cdot142\pm\cdot022$	$+\cdot163\pm\cdot022$	$+\cdot158\pm\cdot022$
Pulse (P) and Respiration (R)	$+\cdot060\pm\cdot022$	$+\cdot081\pm\cdot022$	$+\cdot059\pm\cdot022$

All these correlations are significant, but the sole one of any importance, and even that is of little value, is the relation between temperature and pulse. The only comparative material is that of Körösy reduced by Julia Bell (*Biometrika*, Vol. VIII. p. 232). She found for pulse and respiration $+\cdot108\pm\cdot042$, in excellent agreement with our present value. Her values for pulse and height, $-\cdot072\pm\cdot042$, and for respiration and height, $-\cdot042\pm\cdot042$, are also in accordance with ours considering their probable errors. The smallness of the relation between pulse and respiration is noteworthy, and none of these relations are substantially modified if we make age and weight or age and height constant. In fact temperature, as an examination of Table C shows, has no relation to either height or weight, while greater stature

slightly lessens, greater weight slightly increases the pulse rate. On the other hand the "bantams"—judged by both stature and weight—have a somewhat quicker respiration. But for all practical purposes height and weight may be neglected in considering these physiological variates. It may be of interest to table the actual association of an increase of any one of these variates on the other two.

Probable temperature T_p for a given pulse P: $T_p = 97{\cdot}4336 + {\cdot}01266P$.
Probable temperature T'_p for a given respiration R: $T'_p = 97{\cdot}9860 + {\cdot}02040R$.
Probable pulse P_p for a given temperature T: $P_p = 6{\cdot}5551T - 570{\cdot}6300$*.
Probable pulse P'_p for a given respiration R: $P'_p = 70{\cdot}4986 + {\cdot}19575R$.
Probable respiration R_p for a given temperature T: $R_p = {\cdot}98563T - 77{\cdot}9747$.
Probable respiration R'_p for a given pulse P: $R'_p = 17{\cdot}6291 + {\cdot}01827P$.

DIAGRAM III. Regression Line of Temperature on Pulse for Criminals.



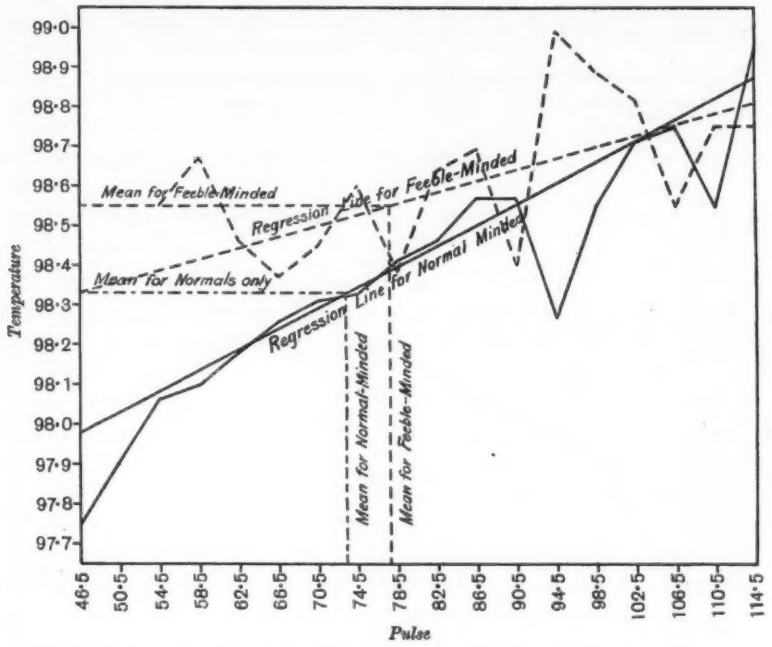
In other words the temperature increases by ·127° for every increase of 10 beats per minute of the pulse and by ·204° for every increase of 10 respirations per minute; the pulse-rate quickens 6·56 beats per minute for each degree of temperature and 1·96 beats per minute for every increase of 10 respirations per minute. Lastly the respiration-rate quickens ·986 respirations per minute for each degree

* For the normal-minded criminals only this line is $P_p = 6{\cdot}4180T - 557{\cdot}7481$.

of temperature, and ·183 respirations per minute for every 10 beats per minute increase of the pulse.

A further interesting point is that, if we could possibly suppose these relations for criminals, whose disorders, if any, do not enforce hospital treatment, to extend beyond the range of values observed, then the pulse would cease at a body temperature $T_m = 87°{\cdot}05$* and the respiration would cease at a body temperature $T'_m = 79°{\cdot}11$. We have no right, however, to extend our *linear* regressions to

DIAGRAM IV. Regression Lines of Temperature on Pulse for Feeble- and Normal-Minded.



these extreme cases; all we can say is that they suggest cessation of pulse before that of respiration, and raise the problem of whether the temperature at death, T_m, is or is not on the average as much below normal temperature as is indicated by the above vague figures.

In Diagram III the regression line of temperature on pulse is drawn to test the general accuracy of linearity in describing this relationship. It will be seen that graphically the accordance is good, the group 45–48 of pulse-rate (see Table XXXIX) has only two entries and the deviation is non-significant. Algebraically for

* Deduced from normal-minded criminals only $T_m = 86°{\cdot}90$.

temperature on pulse $\eta = \cdot 2951$ (without corrections) to be compared with $r = \cdot 2865$ (without corrections) $= \cdot 2881$ after correction.

In Diagram IV the weak-minded have been separated from the normal-minded criminals and the separate regression lines of temperature on pulse are given for both classes. The equations to these lines are:

For normal-minded criminals: $T_p = 97 \cdot 3651 + \cdot 01315 P$,

For weak-minded criminals: $T_p = 97 \cdot 9976 + \cdot 00708 P$,

and the correlations are respectively $\cdot 2906 \pm \cdot 0228$ and $\cdot 1847 \pm \cdot 0471$; thus we see that there is less correlation in the case of the weak-minded and further that the regression lines are no longer, as in the case of temperature on age, parallel. While the weak-minded of low pulse-rate have a higher temperature than the corresponding group of normal-minded, the weak-minded with high pulse-rate tend to have equal or perhaps slightly inferior temperature to the corresponding group of normal-minded.

(5) *On the Correlations between the Physiological and the Physical Variates.* We have already seen that various points of the physique of these criminals were noted by the medical officer. In the first place their capacity for hard labour; actually the class of work on which each criminal was employed was given, but it seemed sufficient for our present purposes to divide these into hard and light labour. In the next place we have the appreciation of muscularity given by the two categories muscular and weak. Physical condition was also marked by the classification fat and thin, corresponding to good and bad state of nutrition. Lastly the general health as appreciated medically was classed as good, indifferent and poor. We made the division Good and Not-good, and used in all cases four-fold tables, where the bi-serial r was not obtainable.

The following correlations of the physical characters were reached:

TABLE D.

Correlations of Variates measuring Physique.

	General Health (Good and Not-good)	Capacity for Labour (Set to Hard or Light Work)	Muscularity (Muscular or Weak)	Nutrition (Fat or Thin)
General Health ...	—	$+ \cdot 770 \pm \cdot 027$	$+ \cdot 639 \pm \cdot 038$	$+ \cdot 451 \pm \cdot 043$
Capacity for labour	$+ \cdot 770 \pm \cdot 027$	—	$+ \cdot 878 \pm \cdot 019$	$+ \cdot 369 \pm \cdot 048$
Muscularity	$+ \cdot 639 \pm \cdot 038$	$+ \cdot 878 \pm \cdot 019$	—	$+ \cdot 199 \pm \cdot 047$
Nutrition (Fat or Thin)	$+ \cdot 451 \pm \cdot 043$	$+ \cdot 369 \pm \cdot 048$	$+ \cdot 199 \pm \cdot 047$	—

It will be seen at once that these correlations are of a very high order, much beyond anything we have hitherto reached. Indeed it is fairly clear that the Medical Officer's general appreciation of health or muscularity tallies closely with

the prison official's appreciation of the type of labour to which the convict can be set. We are justified therefore in considering how far these variates of physique, which clearly have intrinsic value far beyond the difficulties inherent in the personal equation of the medical examiner, are associated with the physiological variates.

TABLE E.

Correlation of Physiological and Physical Variates.

	Temperature	Pulse	Respiration
General Health ...	+·108±·029	−·116±·029	+·025±·029
Capacity for Labour	+·098±·028	−·010±·028	−·098±·028
Muscularity	+·126±·030	−·179±·030	−·130±·030
Nutrition	+·019±·028	+·093±·028	+·096±·028
Height	+·003±·022	−·078±·022	−·144±·022
Weight	+·043±·022	+·114±·022	−·089±·022
Age	−·150±·022	+·121±·022	+·136±·030

Now this table is really very noteworthy for all the correlations are low, even when significant. In other words the general physical condition has very little relation to the physiological variates. In a person, not ill enough for hospital treatment, temperature, pulse, and respiration would hardly be a differential measure of general health, much less of the goodness or badness of the physique in general. It is true that the person in good health has a rather higher temperature and rather lower pulse-rate than one in poor or indifferent health. But there is no significant difference in respiration and the correlations are so low that not only no rough measures of temperature and pulse would aid diagnosis, but really fine numerical determinations would not be of any discriminating value. In capacity for hard labour the pulse plays no part, but it is associated with a slightly higher temperature and a slightly slower respiration. Muscularity is associated with higher temperature, slower pulse and slower respiration. Fatness with higher respiration and pulse, but has no apparent relation to temperature. Height and weight also have no sensible relation to temperature, the larger men have a slower respiration, but the effect of tallness is to slacken pulse, of greater weight to quicken it. Pulse and respiration quicken with age, but temperature falls. In every case, however, these associations are so small that they would be incapable of appreciation except as the mean results of large numbers of accurate records. For ordinary every day experience we can only conclude that nothing can be judged from the physiological variates of physique or from physique of the physiological variates.

(6) *On the Relation of External Temperature to the Physiological Variates.* In the memoir by Williams, Bell and Pearson, it has been shown that such variations as occur in seasonal change of external temperature do not influence the body temperature *of one and the same* individual*. The present material enables us to confirm this result for fairly long series of temperatures on different individuals. We have correlated external temperature with body temperature, pulse and respiration, and find the following values:

Correlation of External Temperature and Body Temperature: $-\cdot002 \pm \cdot023$.
Correlation of External Temperature and Pulse: $+\cdot108 \pm \cdot023$.
Correlation of External Temperature and Respiration: $+\cdot067 \pm \cdot023$.

Thus body and external temperatures are to the extent of seasonal change unrelated. Pulse quickens and respiration quickens in rather warmer weather. But again these environmental influences are of a very unimportant nature, and the associations are even less than those between the physiological and physical characters.

(7) *On the Individuality of the Physiological Variates.* We have seen that temperature, pulse and respiration are variates, which have in reality very little association with physique as measured by such anthropometric characters as age, height or weight; nor are they, within the limits of non-hospital state, at all closely associated with general health or nutrition. Further they are practically unrelated to occupation (nature of labour) or environment (external temperature) as far as we have been hitherto able to measure these factors. Does this therefore signify that they are quantities rapidly varying, changed by a variety of causes not yet appreciated, and of no essential importance to the individual, as having no close relation to any of the bigger measurable factors of personality, physique or environment? The answer to this question is, perhaps, best sought by asking which of all the characters we have dealt with is most closely related to the physiological variates? There cannot be a doubt that the answer must be the mentality of the individual, and this factor of individuality we know to be one that is remarkably persistent and personal. The majority of the weak-minded are congenitally weak-minded, and but few of them really change their intellectual grade in the course of their life-time. Those who believe that intellectual power is the product of environment, or on the other hand that it is closely related to physique, or to any anthropometric measurement of the external body, have yet to demonstrate their position, and many facts have been collected which are markedly opposed to it†. There is much evidence to show that the chief mental characters flow from congenital and hereditary potentialities. Can it be that these physiological variates have a like origin? The actual demonstration of this cannot, perhaps, be completed on the basis of our present data, but very weighty evidence for its truth can, however, be deduced. We have already indicated that in the majority of cases the observation of temperature, pulse and respiration was repeated at an interval

* See *loc. cit.* pp. 5–7.

† See *R. S. Proc.* Vol. 69, pp. 333–342, Vol. 71, pp. 106–114; *Biometrika*, Vol. III. p. 39, et seq., Vol. v. pp. 105–146. Letters in *British Medical Journal*, January 27—March 17, 1906, etc.

varying from 2–3 weeks to 2–3 months. We are thus able to correlate first and second temperatures, first and second pulses and first and second respirations, and the results are of very marked significance. We find the following values of the correlations:

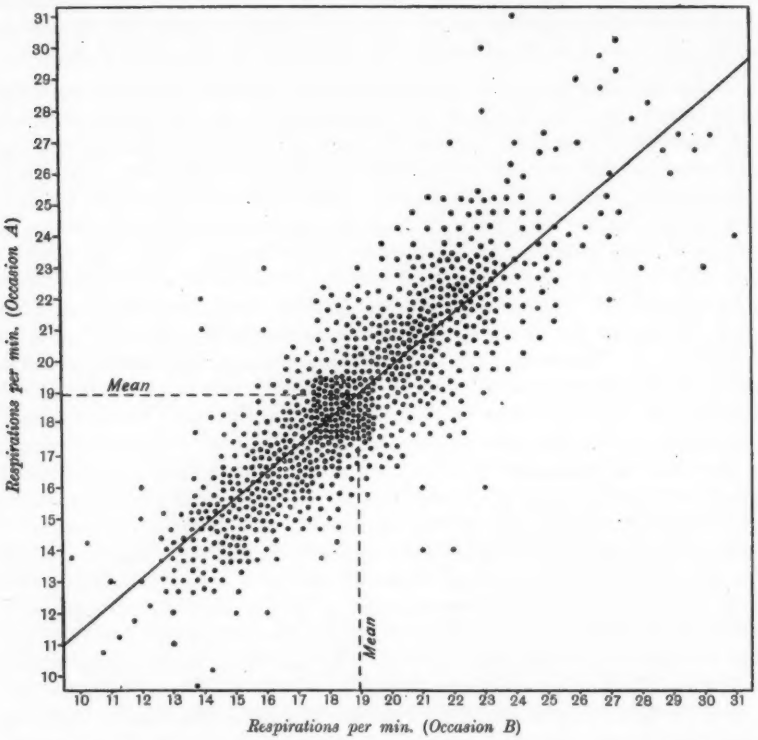
First and Second Temperature: $+ \cdot 395 \pm \cdot 019$*.

First and Second Pulse Rate: $+ \cdot 644 \pm \cdot 012$.

First and Second Respiration Rate: $+ \cdot 856 \pm \cdot 006$.

The accompanying Diagram V indicates the distribution of observations of First and Second Respiration and their clustering about the regression line.

DIAGRAM V. Scatter Diagram and Regression Line for Respirations before and after an interval.



It will be evident at once from the above results that our physiological variates —above all respiration-rate, but markedly also pulse-rate—are essentially constant

* The value found for first and second temperatures in the case of children by Williams, Bell and Pearson (*loc. cit.* p 10) was $+ \cdot 5238 \pm \cdot 0346$.

characters of the individual, characters which change only slightly with age, and which are only moderately linked with each other and with mentality. They are not subject to much influence from environment or occupation, and it appears to us that respiration-rate would probably form as good a mark of individual identity as most anthropometric characters and, since there is a relatively low rate of correlation between these physiological variates, their combined records might be even of value as a supplement* to finger prints in the problem of identification. The above values undoubtedly indicate that the degree of individuality which we find in the external measurable features of the body and in the mental characteristics is maintained, and just possibly exceeded, in the manner in which the various organs of the body carry on their functions; in other words the "individual" is not only characterised by an individualised framework and an individualised controlling faculty for that framework, but its metabolic processes are individualised also. The individual is such not only by his existence in space, but by his manner of existing in time. The individuality in the functioning of organs is as great as the physical differences in the organs themselves. In the future it may well be that heredity will be as effectively studied by measuring the functioning of organs as by measuring the organs themselves. The clinical thermometer and the sygmograph may gain new scientific value when they are introduced into the field of genetics.

I wish, in conclusion, to express my indebtedness to Professor Karl Pearson for his continuous supervision throughout the course of the work and to thank him for many ideas and suggestions set forth in this paper.

* Of course two persons will have the same respiration but hardly an absolute sameness of finger prints. What is meant here is that if finger prints have to be placed in definite descriptive "categories" —a loop of so many ridges or a subclass of arches—these categories will hardly be more individual than the respiration-rate. Again a pulse tracing would correspond for individuality more closely to an actual finger imprint, than a pulse-rate which would represent a certain category of pulse tracings like a finger print class represents a certain type of finger prints.

APPENDIX. CORRELATION TABLES.

TABLE I.

Respirations per minute. × Temperature.

	96·5*	96·7	96·9	97·1	97·3	97·5	97·7	97·9	98·1	98·3	98·5	98·7	98·9	99·1	99·3	99·5	99·7	99·9	Totals
10	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	2
11	—	—	—	—	—	—	—	—	1	1	—	1	—	—	—	—	—	—	3
12	—	—	—	1	—	—	1	3	—	1	1	—	—	—	—	—	—	—	7
13	—	—	—	—	1	2	4	1	2	5	—	1	1	—	1	1	—	—	19
14	—	—	1	1	1	4	6	5	7	6	4	9	2	1	—	—	—	—	47
15	—	1	1	—	3	5	8	7	8	11	8	5	3	6	1	—	—	—	67
16	—	—	—	—	2	5	5	6	14	15	9	12	9	3	2	—	—	—	82
17	—	—	—	1	2	4	2	10	11	13	19	11	7	3	2	—	—	—	85
18	1	—	—	—	6	4	9	10	14	25	16	20	4	7	2	—	—	1	119
19	—	—	—	—	1	2	12	17	15	13	24	12	8	4	3	—	1	—	112
20	—	—	—	—	—	4	5	14	18	15	8	14	4	2	1	—	—	—	85
21	—	—	—	—	1	1	4	8	10	14	20	13	9	4	3	2	—	—	89
22	—	1	—	—	1	2	4	3	8	12	13	11	11	3	2	—	—	—	71
23	—	—	—	—	1	1	6	7	6	8	6	6	8	3	1	1	1	—	55
24	—	—	—	—	3	—	3	1	2	3	3	6	1	3	—	1	—	—	26
25	—	—	—	—	—	—	—	3	4	4	4	6	2	2	—	1	—	—	26
26	—	—	—	—	—	—	—	—	1	1	1	1	1	—	—	1	—	—	6
27	—	—	—	—	—	—	1	—	2	3	2	1	1	—	—	—	—	—	10
28	—	—	—	—	—	—	—	1	—	2	—	1	1	—	—	—	—	—	5
29	—	—	—	—	—	—	2	—	1	—	—	—	1	—	—	—	—	—	4
30	—	—	—	—	—	—	—	1	—	—	1	1	—	—	—	—	—	—	3
31	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
Totals	1	2	2	3	22	34	72	97	124	153	140	132	73	41	18	7	2	1	924

TABLE II. General Health.	Good ...	0	1	2	3	13	24	41	59	81	101	103	95	47	27	13	6	3	1	620
	Poor ...	1	1	—	—	9	10	31	37	44	49	35	36	24	13	4	1	—	—	295
TABLE III. Nutrition.	Thin ...	1	1	—	2	11	18	47	54	68	86	78	82	37	27	9	5	1	—	527
	Fat ...	—	1	2	1	11	16	25	42	57	64	60	49	34	13	8	2	2	1	388
TABLE IV. Muscularity.	Muscular	—	1	2	1	16	23	55	67	92	108	117	106	53	34	13	5	3	1	697
	Weak	1	1	—	2	6	11	17	29	33	42	21	25	18	6	4	2	—	—	218
	Totals for each Table	1	2	2	3	22	34	72	96	125	150	138	131	71	40	17	7	3	1	915

* 96·5 and 96·6, the temperatures were only read to tenths.

* 96·5 and 96·6, the temperatures were only read to tenths.

TABLE V.

Respirations per minute.	Pulse-Beats per minute. 45—48	49—52	53—56	57—60	61—64	65—68	69—72	73—76	77—80	81—84	85—88	89—92	93—96	97—100	101—104	105—108	109—112	113—116	Totals
10	—	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	2
11	—	—	—	—	—	—	—	2	1	—	—	—	—	—	—	—	—	—	3
12	—	—	—	1	1	3	2	—	—	—	—	—	—	—	—	—	—	—	7
13	—	—	1	2	2	2	3	3	3	1	1	1	—	—	—	—	—	—	19
14	—	—	1	4	7	13	7	7	2	2	1	—	—	—	—	1	1	1	47
15	1	—	2	9	4	20	8	9	5	3	3	1	1	—	1	1	—	—	68
16	—	2	2	4	15	13	10	12	8	4	3	4	2	1	1	—	—	1	82
17	—	—	2	6	9	13	14	11	7	11	5	2	2	—	1	—	2	—	85
18	1	2	—	11	13	19	15	15	12	13	10	2	3	2	—	1	—	—	119
19	—	—	1	8	6	19	17	18	17	10	8	3	1	2	2	—	—	—	112
20	—	—	3	2	6	10	15	14	10	6	9	7	1	1	1	—	—	—	85
21	—	—	2	2	14	9	8	15	12	8	7	6	—	5	—	1	—	—	89
22	—	—	1	3	2	18	2	10	10	15	4	4	—	2	—	—	—	—	71
23	—	—	—	4	6	6	6	5	7	2	6	7	2	4	—	—	—	—	55
24	—	—	—	—	1	2	4	5	6	2	1	2	2	—	—	—	—	—	25
25	—	—	—	—	1	2	6	4	4	2	1	3	—	—	1	1	—	1	26
26	—	—	—	—	—	—	1	—	1	1	1	—	1	—	1	—	—	—	6
27	—	—	1	1	1	—	1	1	2	3	—	—	—	—	—	—	—	—	10
28	—	—	—	—	1	—	1	—	—	1	1	—	—	—	1	—	—	—	5
29	—	1	1	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	4
30	—	—	—	—	—	1	—	—	1	—	1	—	—	—	—	—	—	—	3
31	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
Totals	2	5	17	57	90	150	120	131	109	86	62	42	15	18	9	5	3	3	924

TABLE VI. Good Health.	Good ...	1	4	9	41	66	109	80	93	71	57	37	24	7	9	5	4	1	2	620
	Poor ...	1	1	8	16	24	40	36	39	37	27	23	18	7	9	4	1	2	1	294
TABLE VII. Nutrition.	Thin ...	—	2	12	32	60	77	64	62	62	48	42	28	10	10	6	5	3	3	526
	Fat ...	2	3	5	25	30	72	52	70	46	36	18	14	4	8	3	—	—	—	388
TABLE VIII. Muscularity.	Muscular	1	5	13	45	75	126	98	95	83	56	36	25	9	14	8	5	1	2	697
	Weak ...	1	—	4	12	15	23	18	37	25	28	24	17	5	4	1	—	2	1	217
	Totals for each Table	2	5	17	57	90	149	116	132	108	84	60	42	14	18	9	5	3	3	914

TABLE IX.

Respirations per minute (Occasion A).

Respirations per minute (Occasion B).	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	Totals
10	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
11	—	2	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
12	—	—	2	1	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5
13	—	1	1	6	8	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	19
14	2	—	—	8	8	16	5	—	2	—	—	1	1	—	—	—	—	—	—	—	—	—	43
15	—	—	1	3	16	14	15	8	5	—	—	—	—	—	—	—	—	—	—	—	—	—	62
16	—	—	1	—	5	15	20	18	7	4	—	1	—	1	—	—	—	—	—	—	—	—	72
17	—	—	—	—	—	8	18	22	19	9	7	—	—	—	—	—	—	—	—	—	—	—	83
18	—	—	—	—	2	5	7	19	32	31	7	5	5	—	—	—	—	—	—	—	—	—	113
19	—	—	—	—	—	—	4	9	31	24	15	11	6	1	—	—	—	—	—	—	—	—	101
20	—	—	—	—	—	—	—	7	7	15	20	16	5	4	2	—	—	—	—	—	—	—	76
21	—	—	—	—	1	—	1	—	5	11	16	20	12	9	2	2	—	—	—	—	—	—	79
22	—	—	—	—	1	—	—	—	5	6	5	12	18	13	4	3	—	1	—	—	—	—	68
23	—	—	—	—	—	—	1	—	—	1	4	9	13	12	5	6	—	—	1	—	1	—	53
24	—	—	—	—	—	—	—	—	—	—	2	2	4	5	2	4	3	1	—	—	—	1	24
25	—	—	—	—	—	—	—	—	—	—	—	2	3	6	4	2	—	3	—	—	—	—	20
26	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3	—	—	1	—	1	—	—	5
27	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	3	1	—	—	2	2	—	10
28	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	2	—	—	—	3
29	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2	—	—	—	—	3
30	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	2	—	—	—	—	3
31	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
Totals	2	3	5	19	43	62	72	83	113	101	76	79	68	53	24	20	5	10	3	3	3	1	848

TABLE X. General Health.	Good ...	1	2	6	12	30	40	53	59	89	84	51	52	47	36	15	19	2	9	4	3	3	1	618
	Poor ...	1	1	1	7	18	27	27	25	29	26	34	35	22	18	10	7	4	1	1	1	—	—	295
TABLE XI. Nutrition.	Thin ...	1	1	4	8	25	36	46	48	63	62	57	46	41	36	16	15	5	5	4	3	2	1	525
	Fat ...	1	2	3	11	23	31	34	36	55	48	28	41	28	18	9	11	1	5	1	1	1	—	388
TABLE XII. Muscularity.	Muscular	2	3	7	16	34	51	74	66	89	83	62	65	50	43	14	16	2	7	4	3	3	1	695
	Weak ...	—	—	—	3	14	16	6	18	29	27	23	22	19	11	11	10	4	3	1	1	—	—	218
	Totals for each Table	2	3	7	19	48	67	80	84	118	110	85	87	69	54	25	26	6	10	5	4	3	1	913

TABLE XIII.

Temperature (Occasion A).

TABLE XIII.

Temperature (Occasion A).

Temperature (Occasion B).	96·5	96·7	96·9	97·1	97·3	97·5	97·7	97·9	98·1	98·3	98·5	98·7	98·9	99·1	99·3	99·5	99·7	99·9	Totals
96·5—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
96·7—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	2
96·9—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
97·1—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	2
97·3—	—	—	—	—	2	2	2	5	2	4	—	1	2	—	—	—	—	—	20
97·5—	—	—	—	1	2	4	4	8	3	7	2	1	1	—	—	—	—	—	33
97·7—	—	—	—	—	2	4	20	11	7	9	7	4	1	1	—	1	—	—	67
97·9—	—	—	—	—	5	8	11	24	14	10	7	6	4	—	1	—	—	—	90
98·1—	—	1	—	1	2	3	7	14	34	19	17	13	3	3	1	—	—	1	119
98·3—	—	1	—	—	4	7	9	10	19	28	23	22	14	4	2	—	—	—	143
98·5—	—	—	1	—	—	2	7	7	17	23	36	21	5	8	5	—	—	—	132
98·7—	1	—	—	—	1	1	4	6	13	22	21	26	8	11	2	2	2	—	120
98·9—	—	—	—	—	2	1	1	4	3	14	5	8	18	2	5	2	—	—	65
99·1—	—	—	—	—	—	—	1	—	3	4	8	11	2	2	1	—	—	—	32
99·3—	—	—	—	—	—	—	—	1	1	2	5	2	5	1	—	—	1	—	18
99·5—	—	—	—	—	—	—	1	—	—	—	—	2	2	—	—	—	—	—	5
99·7—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	1	—	—	—	3
99·9—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
Totals	1	2	1	2	20	33	67	90	119	143	132	120	65	32	18	5	3	1	854

TABLE XIV.

Prison Labour.

General Health.	Hard	Light	Totals
Good	260	75	335
Poor	33	124	157
Totals	293	199	492

TABLE XV.

Muscularity.

	Musc.	Weak	Totals
Good	298	37	335
Poor	81	77	158
Totals	379	114	493

TABLE XVI.

Muscularity.

Prison Labour.	Musc.	Weak	Totals
Hard	286	7	293
Light	93	106	199
Totals	379	113	492

TABLE XVII.

Height in inches.

Weight in lbs.	58—	59—	60—	61—	62—	63—	64—	65—	66—	67—	68—	69—	70—	71—	72—	73—	74—	75—	76—	Totals
90—99	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
100—109	—	1	—	—	1	1	—	—	—	—	1	—	—	—	—	—	—	—	—	4
110—119	1	—	5	9	5	6	3	1	—	—	1	—	—	—	—	—	—	—	—	31
120—129	1	2	3	7	15	12	13	5	8	2	1	—	—	—	—	—	—	—	—	69
130—139	—	—	2	14	12	25	30	19	12	8	5	1	—	—	—	—	—	—	—	128
140—149	—	1	4	5	3	13	21	18	22	18	6	6	2	1	—	—	—	—	—	120
150—159	—	—	—	—	1	6	2	10	11	14	10	7	4	1	2	—	—	—	—	68
160—169	—	—	—	1	2	—	1	4	3	12	7	6	4	—	—	—	—	—	—	40
170—179	—	—	—	1	—	—	1	3	2	5	2	3	3	1	—	—	—	—	—	21
180—189	—	—	—	—	—	—	—	—	1	1	—	1	1	—	—	—	—	—	—	4
190—199	—	—	—	—	—	—	1	—	—	—	—	1	1	—	—	—	—	—	—	3
200—209	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
210—219	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
220—229	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2

TABLE XVIII.
Mentality.

Normal ...	1	2	10	29	30	47	61	54	49	42	28	23	14	2	2	—	—	—	1	395
Weak-Minded	1	2	4	8	11	16	11	6	10	18	5	3	1	1	—	—	—	—	—	97
Totals for each Table	2	4	14	37	41	63	72	60	59	60	33	26	15	3	2	—	—	—	1	492

TABLE XIX.

Pulse (Occasion A).

Pulse (Occasion B).	45—48	49—52	53—56	57—60	61—64	65—68	69—72	73—76	77—80	81—84	85—88	89—92	93—96	97—100	101—104	105—108	109—112	113—116	Totals
45—48	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
49—52	1	—	2	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	5
53—56	—	2	2	6	1	3	1	2	—	—	—	—	—	—	—	—	—	—	17
57—60	1	1	6	12	17	7	1	2	3	2	—	—	—	—	—	—	—	—	52
61—64	—	1	1	17	16	20	14	7	2	3	1	—	—	—	—	—	—	—	82
65—68	—	—	3	7	20	42	27	14	14	6	5	1	2	—	—	—	—	—	141
69—72	—	—	1	1	14	27	28	18	12	5	2	—	—	—	—	1	—	—	109
73—76	—	—	2	2	7	14	18	36	15	8	5	7	2	2	2	1	—	—	121
77—80	—	—	—	3	2	14	12	15	26	20	5	4	—	1	—	—	—	—	102
81—84	—	—	—	2	3	6	5	8	20	16	14	3	—	2	—	—	—	—	79
85—88	—	—	—	—	1	5	2	5	5	14	12	9	1	1	1	2	1	1	60
89—92	—	—	—	—	—	1	—	7	4	3	9	6	3	3	1	—	1	—	38
93—96	—	—	—	—	—	2	—	2	—	—	1	3	2	—	2	—	—	—	12
97—100	—	—	—	—	—	—	—	2	1	2	1	3	—	4	1	1	1	—	16
101—104	—	—	—	—	—	—	—	2	—	—	1	1	2	1	—	—	—	—	7
105—108	—	—	—	—	—	—	1	1	—	—	2	—	—	1	—	—	—	—	5
109—112	—	—	—	—	—	—	—	—	—	—	1	1	—	1	—	—	—	—	3
113—116	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
Totals	2	5	17	52	82	141	109	121	102	79	60	38	12	16	7	5	3	1	952

TABLE XX.

Age.

115—119																						
Totals	2	5	17	52	82	141	109	121	102	79	60	38	12	16	7	5	3	1	952

TABLE XX.

Age.

Weight in lbs.	21—	24—	27—	30—	33—	36—	39—	42—	45—	48—	51—	54—	57—	60—	63—	66—	69—	72—	75—	78—	81—	84—	Totals
90—99	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
100—109	—	—	1	1	—	—	—	—	—	—	—	1	—	—	—	—	1	—	—	—	—	—	4
110—119	—	5	5	3	2	2	2	4	1	1	—	—	—	2	—	1	1	1	1	—	—	—	31
120—129	3	7	4	12	4	10	1	1	3	3	2	2	3	2	4	2	1	3	1	1	—	—	69
130—139	2	8	12	15	16	7	13	10	6	7	5	5	1	1	8	6	2	3	1	—	—	—	128
140—149	3	9	13	16	14	12	10	5	3	2	5	6	5	5	6	2	2	3	—	—	—	—	121
150—159	2	1	11	2	8	7	8	8	5	—	1	1	2	1	3	3	2	1	—	1	1	1	68
160—169	—	2	—	7	4	2	5	5	1	4	2	—	1	4	1	1	1	—	—	—	—	—	40
170—179	—	2	—	1	—	2	3	—	2	—	1	2	1	4	1	—	1	—	—	1	—	—	21
180—189	—	—	—	—	—	1	—	—	—	—	—	—	1	1	1	—	—	—	—	—	—	—	4
190—199	—	—	—	—	—	—	—	—	—	1	—	—	—	1	1	—	—	—	—	—	—	—	3
200—209	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
210—219	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
220—229	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1	—	—	—	2
Totals	10	34	46	57	48	43	42	33	21	18	17	18	14	21	25	15	11	11	4	3	1	1	493

TABLE XXI.

Height in inches.																							
58—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
59—	—	—	—	2	—	—	1	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	4
60—	2	2	2	—	—	1	2	1	1	—	—	—	—	2	—	—	—	1	—	—	—	—	14
61—	—	2	2	3	4	3	6	6	2	1	—	—	1	3	1	—	1	1	—	1	—	—	37
62—	—	2	3	4	5	7	1	—	3	4	2	2	1	2	1	2	1	1	—	—	—	—	41
63—	—	6	4	13	8	4	2	5	3	—	2	3	3	2	4	3	—	—	1	—	—	—	63
64—	3	7	7	9	6	3	4	3	1	3	4	4	—	2	8	2	1	3	2	—	—	—	72
65—	2	4	9	6	6	4	5	4	1	4	1	2	—	—	3	3	2	3	—	1	—	—	60
66—	1	2	5	4	5	8	9	2	2	2	1	3	5	—	2	3	2	1	—	1	1	—	59
67—	—	3	8	4	4	6	5	5	3	—	5	—	1	7	5	1	1	1	—	—	—	1	36
68—	—	1	4	5	2	3	3	4	—	2	2	2	2	1	—	1	1	—	—	—	—	—	33
69—	1	3	2	4	4	2	3	1	1	2	—	1	—	2	—	—	—	—	—	—	—	—	26
70—	—	1	—	—	2	2	1	2	4	—	—	4	1	—	1	—	—	—	—	—	—	—	15
71—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	3
72—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
73—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
74—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
75—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
76—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
Totals	10	34	46	56	48	43	42	33	21	18	17	18	14	21	25	15	11	11	4	3	1	1	492

UNIVERSITY OF WINDSOR LIBRARY
131948

TABLE XXII.

Weight in lbs.	Temperature.																	
	96·5	*96·7*	*96·9*	*97·1*	*97·3*	*97·5*	*97·7*	*97·9*	*98·1*	*98·3*	*98·5*	*98·7*	*98·9*	*99·1*	*99·3*	*99·5*	*99·7*	*99·9*	Totals
90—99	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	—	2
100—109	—	—	—	—	—	—	—	2	2	1	—	2	1	—	—	—	—	—	8
110—119	—	—	—	—	1	—	3	7	9	7	9	12	9	2	—	—	—	—	59
120—129	—	1	—	—	6	4	6	17	13	27	2[illegible]	16	9	3	3	1	1	—	128
130—139	—	—	1	—	6	6	23	22	33	38	39	28	21	13	7	1	1	—	239
140—149	1	—	—	2	6	12	14	27	27	37	26	32	16	11	5	3	—	1	220
150—159	—	1	—	1	2	8	11	9	16	21	24	21	6	4	1	—	1	—	126
160—169	—	—	—	—	—	1	11	5	14	7	15	10	6	3	—	1	—	—	73
170—179	—	—	1	—	1	2	2	6	4	7	3	6	2	4	1	1	—	—	40
180—189	—	—	—	—	—	1	1	1	2	1	—	2	—	—	—	—	—	—	8
190—199	—	—	—	—	—	—	1	—	1	3	—	1	—	—	—	—	—	—	6
200—209	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	2
210—219	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
220—229	—	—	—	—	—	—	—	—	3	—	1	—	—	—	—	—	—	—	4
Totals	1	2	2	3	22	34	72	96	125	150	138	131	71	40	17	7	3	1	915

TABLE XXIII.

Height in inches.																		
58—	—	—	—	—	1	—	—	—	—	—	1	—	2	—	—	—	—	—	4
59—	—	—	—	—	—	—	—	2	—	2	1	1	2	—	—	—	—	—	8
60—	—	—	—	—	1	1	—	1	5	5	2	6	3	1	1	—	1	—	27
61—	—	—	—	—	2	3	3	8	8	7	7	11	8	6	2	1	—	—	66
62—	—	—	—	—	1	1	6	7	14	16	12	10	7	2	1	—	—	—	77
63—	—	1	—	1	6	4	8	11	22	19	20	18	7	2	2	—	—	—	121
64—	1	—	—	—	2	6	14	17	14	26	19	14	8	7	2	1	1	—	132
65—	—	—	—	1	4	6	8	18	13	20	13	12	10	3	4	—	—	—	112
66—	—	—	—	—	1	6	15	12	8	18	23	17	7	3	1	2	—	—	113
67—	—	1	—	—	2	2	10	13	11	18	19	19	5	6	1	1	1	1	110
68—	—	—	1	—	1	2	3	3	17	6	8	11	4	3	1	—	—	—	60
69—	—	—	—	—	1	1	4	2	6	9	8	5	5	3	1	2	—	—	47
70—	—	—	1	1	—	—	1	2	3	3	4	5	2	3	—	—	—	—	25
71—	—	—	—	—	—	—	—	—	2	1	—	—	1	1	1	—	—	—	6
72—	—	—	—	—	—	2	—	—	—	—	1	1	—	—	—	—	—	—	4
73—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
74—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
75—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
76—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	2
Totals	1	2	2	3	22	34	72	96	125	150	138	130	71	40	17	7	3	1	914

TABLE XXIV.

TABLE XXIV.

Respirations per minute.

Weight in lbs.	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	Totals
90—99	—	—	—	—	1	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	2
100—109	—	—	—	—	—	—	—	2	2	—	3	—	—	—	1	—	—	—	—	—	—	—	8
110—119	—	—	—	1	2	2	5	3	10	8	4	4	4	8	3	2	—	1	1	—	1	—	59
120—129	—	—	—	3	2	5	11	18	16	12	15	9	14	11	4	2	2	—	—	1	1	—	126
130—139	1	—	4	2	20	19	25	17	29	30	19	22	15	11	5	10	2	5	—	2	1	—	239
140—149	—	—	1	3	10	20	19	18	32	29	21	25	13	14	3	8	—	1	3	—	—	—	220
150—159	1	1	2	3	5	13	12	13	14	14	12	13	9	3	3	2	1	2	1	1	—	1	126
160—169	—	2	—	1	7	2	3	6	6	9	7	10	7	5	4	2	1	1	—	—	—	—	73
170—179	—	—	—	5	—	3	2	5	5	6	4	4	4	—	2	—	—	—	—	—	—	—	40
180—189	—	—	—	—	—	—	2	1	3	—	—	—	1	1	—	—	—	—	—	—	—	—	8
190—199	—	—	—	—	—	2	—	1	1	2	—	—	—	—	—	—	—	—	—	—	—	—	6
200—209	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
210—219	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
220—229	—	—	—	—	1	—	1	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	4
Totals	2	3	7	19	48	67	80	84	118	110	85	87	69	54	25	26	6	10	5	4	3	1	913

TABLE XXV.

Height in inches.	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	Totals
58—	—	—	—	—	—	—	1	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
59—	—	—	—	—	—	—	—	1	2	1	2	—	—	—	—	1	—	1	—	—	—	—	8
60—	—	—	—	1	1	—	4	3	4	3	2	3	2	2	1	—	—	—	—	—	—	—	26
61—	—	—	—	—	3	2	5	4	8	3	8	10	5	7	3	7	1	—	—	—	—	—	66
62—	—	—	—	2	4	6	5	7	5	8	7	8	6	9	3	1	—	2	—	2	1	—	76
63—	1	—	—	2	5	6	11	12	22	17	4	9	10	7	4	5	—	2	1	—	2	1	121
64—	—	—	4	2	7	10	9	13	23	17	17	6	8	8	4	3	—	1	—	—	—	—	132
65—	—	—	1	1	8	12	10	11	8	13	12	15	7	3	3	3	1	1	2	1	—	—	112
66—	—	—	—	3	5	12	13	5	18	19	10	9	11	4	1	2	—	1	—	—	—	—	113
67—	1	—	—	1	5	5	12	5	12	16	8	11	10	9	5	4	3	1	1	1	—	—	110
68—	—	3	—	4	7	3	5	13	9	2	5	7	2	—	—	—	—	—	—	—	—	—	60
69—	—	—	—	3	1	4	3	2	4	10	4	8	3	4	—	—	—	—	1	—	—	—	47
70—	—	—	2	—	—	4	2	3	3	—	5	1	4	—	1	—	—	—	—	—	—	—	25
71—	—	—	—	—	1	2	—	1	—	—	—	—	—	—	—	—	1	1	—	—	—	—	6
72—	—	—	—	—	1	1	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	4
73—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
74—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
75—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
76—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	2
Totals	2	3	7	19	48	67	80	84	118	110	84	87	69	54	25	26	6	10	5	4	3	1	912

TABLE XXVI.

Pulse.

Weight in lbs.	45—48	49—52	53—56	57—60	61—64	65—68	69—72	73—76	77—80	81—84	85—88	89—92	93—96	97—100	101—104	105—108	109—112	113—116	Totals
90—99	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	2
100—109	—	—	—	—	—	1	1	1	1	1	3	—	—	—	—	—	—	—	8
110—119	—	—	—	1	3	7	6	6	7	10	5	5	1	3	1	1	—	2	58
120—129	—	—	2	9	18	17	14	12	17	10	11	7	3	3	3	—	2	—	128
130—139	—	1	4	23	24	39	28	29	29	22	16	8	5	5	1	4	—	1	239
140—149	—	1	8	16	22	41	25	43	14	22	12	9	3	1	3	—	—	—	220
150—159	1	2	1	5	15	26	16	15	19	8	4	7	—	5	1	—	1	—	126
160—169	—	1	—	2	6	6	15	11	15	5	7	3	1	1	—	—	—	—	73
170—179	—	—	2	—	1	6	7	13	4	3	1	3	—	—	—	—	—	—	40
180—189	—	—	—	—	—	2	2	—	2	2	—	—	—	—	—	—	—	—	8
190—199	1	—	—	1	1	1	—	—	—	—	1	—	1	—	—	—	—	—	6
200—209	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	2
210—219	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
220—229	—	—	—	—	—	2	2	—	—	—	—	—	—	—	—	—	—	—	4
Totals	2	5	17	57	90	149	116	132	108	84	60	42	14	18	9	5	3	3	914

TABLE XXVII.

Height in inches.	58—	59—	60—	61—	62—	63—	64—	65—	66—	67—	68—	69—	70—	71—	72—	73—	74—	75—	76—	Totals
	—	—	—	—	—	1	—	—	2	1	—	—	—	—	—	—	—	—	—	4
	—	—	—	1	1	1	1	—	1	1	2	—	—	—	—	—	—	—	—	8
	—	—	—	—	—	9	1	4	3	5	3	2	—	—	—	—	—	—	—	27
	—	—	3	8	4	11	3	5	13	3	2	5	2	3	1	1	—	1	—	65
	—	1	2	6	14	8	12	13	5	5	4	3	2	—	1	—	1	—	—	77
	—	—	—	6	10	16	15	18	11	18	14	6	1	4	2	—	—	—	—	121
	1	3	2	10	18	25	14	17	13	12	5	5	1	2	—	2	1	1	—	132
	—	—	1	6	11	17	17	23	11	7	4	5	6	2	1	1	—	—	—	112
	—	1	4	9	10	15	15	18	16	10	7	4	1	1	1	1	—	—	—	113
	—	—	1	4	8	17	16	18	14	8	7	9	—	5	2	—	1	—	—	110
	—	—	1	3	6	11	9	8	7	5	6	1	—	1	1	—	—	1	—	60
	1	—	2	3	5	12	8	5	5	2	4	—	—	—	—	—	—	—	—	47
	—	—	1	—	1	1	4	2	5	7	2	1	1	—	—	—	—	—	—	25
	—	—	—	1	1	1	—	—	2	—	—	1	—	—	—	—	—	—	—	6
	—	—	—	—	1	2	1	—	—	—	—	—	—	—	—	—	—	—	—	4
	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	2
Totals	2	5	17	57	90	149	116	131	108	84	60	42	14	18	9	5	3	3		913

TABLE XXVIII.

Pulse.

Totals	2	5	17	57	90	145	115	131	105	84	61	42	14	18	9	5	3	3	926

TABLE XXVIII.

Age.	Pulse. 45—48	49—52	53—56	57—60	61—64	65—68	69—72	73—76	77—80	81—84	85—88	89—92	93—96	97—100	101—104	105—108	109—112	113—116	Totals
21—23	—	—	—	1	2	2	3	3	1	2	2	1	—	—	—	—	—	—	17
24—26	—	—	1	6	5	9	9	12	6	5	4	1	1	3	1	—	—	—	63
27—29	1	2	2	7	15	16	10	5	6	7	5	3	1	1	1	—	1	1	84
30—32	—	1	3	7	9	25	17	17	5	7	6	3	1	3	—	3	—	—	107
33—35	—	—	1	8	4	17	12	16	15	7	3	5	—	1	—	—	—	—	89
36—38	—	—	1	3	11	13	16	12	9	6	4	3	2	—	1	—	—	—	81
39—41	—	—	—	5	11	11	10	12	14	6	4	1	1	2	—	1	1	—	79
42—44	—	—	1	3	3	10	4	6	7	9	7	—	3	4	2	—	—	2	61
45—47	—	—	1	3	5	7	4	2	7	5	3	1	—	—	—	—	—	—	38
48—50	—	—	1	2	5	8	3	3	4	1	1	3	2	—	—	—	—	—	33
51—53	—	—	—	—	3	3	3	8	6	3	2	1	—	—	1	—	—	—	30
54—56	—	—	1	4	2	1	4	8	3	4	3	2	1	1	—	—	—	—	34
57—59	—	1	1	—	3	1	1	2	4	4	3	3	1	—	2	—	—	—	26
60—62	1	1	1	2	3	6	9	7	2	1	3	4	—	1	—	—	—	—	41
63—65	—	—	—	1	2	10	7	7	5	7	4	5	1	—	—	1	1	—	51
66—68	—	—	1	5	2	4	2	3	3	4	3	1	1	—	1	—	—	—	30
69—71	—	—	—	—	—	1	4	4	6	3	3	2	—	2	—	—	—	—	25
72—74	—	—	2	—	4	3	1	2	4	3	1	1	—	—	—	—	—	—	21
75—77	—	—	—	—	—	3	1	—	—	1	1	1	—	—	—	—	—	—	7
78—80	—	—	—	—	—	—	—	1	2	—	1	1	—	—	—	—	—	—	5
81—83	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	2
84—86	—	—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	2
Totals	2	5	17	57	90	150	120	132	109	86	63	42	15	18	9	5	3	3	926

TABLE XXIX.

Mentality.																			
Weak-Minded	—	—	1	10	16	19	21	29	25	20	17	12	5	8	3	3	1	1	191
Normal ...	2	5	16	47	74	131	99	103	84	66	46	30	10	10	6	2	2	2	735
Totals	2	5	17	57	90	150	120	132	109	86	63	42	15	18	9	5	3	3	926

TABLE XXX.

Labour.																			
Hard ...	1	3	7	36	54	99	77	80	61	41	28	16	8	11	6	3	—	2	533
Light ...	1	2	10	21	36	50	38	52	47	42	32	26	6	7	3	2	3	1	379
Totals	2	5	17	57	90	149	115	132	108	83	60	42	14	18	9	5	3	3	912

TABLE XXXI.

Respirations per minute.

Age.	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	Totals
21—23	—	—	—	—	—	—	1	3	2	3	2	3	1	—	1	1	—	—	—	—	—	—	17
24—26	—	—	2	3	—	1	7	6	7	10	4	4	8	7	1	1	—	1	—	—	—	—	62
27—29	1	—	—	3	5	5	11	11	10	10	8	9	4	2	2	3	—	—	1	—	—	—	85
30—32	—	2	—	—	6	16	6	15	17	16	8	6	8	1	1	3	—	1	—	—	1	—	107
33—35	—	—	—	—	5	7	8	5	11	11	7	9	7	8	3	4	1	2	—	—	1	—	89
36—38	—	—	2	1	1	6	12	4	15	11	9	5	2	7	5	—	1	—	—	—	—	—	81
39—41	1	1	—	4	6	7	9	6	8	7	4	9	3	4	2	3	1	1	2	1	—	—	79
42—44	—	—	—	—	4	4	6	5	8	11	6	8	5	2	1	—	—	—	—	—	—	1	61
45—47	—	—	1	—	1	1	3	3	3	3	6	2	6	2	2	1	—	1	—	1	1	—	37
48—50	—	—	—	—	1	3	2	2	4	4	2	7	3	4	—	1	—	—	—	—	—	—	33
51—53	—	—	—	3	3	3	2	2	3	4	—	4	2	2	—	2	—	—	—	—	—	—	30
54—56	—	—	1	1	2	4	4	4	7	2	2	2	1	3	—	—	—	—	1	—	—	—	34
57—59	—	—	—	1	1	1	4	5	—	3	3	6	1	—	—	—	1	—	—	—	—	—	26
60—62	—	—	—	1	2	1	—	4	6	3	6	4	5	5	1	1	—	1	—	1	—	—	41
63—65	—	—	1	2	6	5	2	4	9	3	3	5	4	4	1	1	—	1	—	—	—	—	51
66—68	—	—	—	—	3	2	4	1	4	2	3	2	2	1	2	1	1	1	—	1	—	—	30
69—71	—	—	—	—	—	—	—	2	2	2	7	3	4	1	1	1	1	1	—	—	—	—	25
72—74	—	—	—	—	2	1	1	3	3	2	3	—	2	—	2	2	—	—	—	—	—	—	21
75—77	—	—	—	—	—	—	—	—	—	2	—	—	1	2	1	—	—	—	1	—	—	—	7
78—80	—	—	—	—	—	—	—	—	—	3	1	—	—	—	—	1	—	—	—	—	—	—	5
81—83	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	2
84—86	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	2
Totals	2	3	7	19	48	67	82	85	119	112	85	89	71	55	26	26	6	10	5	4	3	1	925

TABLE XXXII.

Mentality.																							
Weak-Minded	2	—	—	2	8	8	15	18	20	25	16	22	16	14	6	5	3	6	2	1	1	—	190
Normal ...	—	3	7	17	40	59	67	67	99	87	69	67	55	41	20	21	3	4	3	3	2	1	735
Totals	2	3	7	19	48	67	82	85	119	112	85	89	71	55	26	26	6	10	5	4	3	1	925

TABLE XXXIII.

Labour.																							
Hard ...	1	2	6	9	21	34	60	52	74	69	45	51	39	30	13	13	2	4	4	—	2	1	532
Light ...	1	1	1	9	26	33	20	32	44	41	40	36	30	24	12	13	4	6	1	4	1	—	379
Totals	2	3	7	18	47	67	80	84	118	110	85	87	69	54	25	26	6	10	5	4	3	1	911

Totals	2	3	7	18	47	67	80	84	118	110	85	87	69	54	25	26	6	10	5	4	3	1	911

TABLE XXXIV.

Age.	Temperature. 96·5	96·7	96·9	97·1	97·3	97·5	97·7	97·9	98·1	98·3	98·5	98·7	98·9	99·1	99·3	99·5	99·7	99·9	Totals
21—23	—	—	—	—	1	—	1	—	2	—	3	4	3	2	—	1	—	—	17
24—26	—	—	—	1	1	3	6	5	6	9	11	8	5	4	3	—	1	—	63
27—29	—	—	1	—	5	4	4	9	10	14	15	14	3	5	—	—	1	—	85
30—32	—	—	—	—	2	5	4	8	15	15	23	17	7	7	4	—	—	—	107
33—35	—	—	—	—	2	4	5	7	9	12	21	14	7	4	1	2	1	—	89
36—38	—	—	—	—	3	—	6	9	10	13	12	14	7	5	2	—	—	—	81
39—41	—	1	—	—	—	2	5	9	7	12	14	9	12	6	1	1	—	—	79
42—44	—	—	—	—	2	—	6	7	8	12	8	9	5	2	1	1	—	—	61
45—47	—	—	—	—	—	1	1	5	8	7	4	8	3	—	1	—	—	—	38
48—50	—	—	—	—	—	1	5	4	5	7	3	4	2	2	—	—	—	—	33
51—53	—	—	—	—	—	5	5	4	2	5	1	4	3	—	—	—	—	1	30
54—56	—	—	1	—	4	—	1	6	6	7	1	2	2	1	3	—	—	—	34
57—59	—	—	—	—	—	—	5	—	5	5	4	4	1	—	1	1	—	—	26
60—62	—	—	—	1	—	3	4	5	5	6	7	3	5	2	—	—	—	—	41
63—65	—	1	—	—	1	1	2	7	8	12	6	9	3	1	—	—	—	—	51
66—68	1	—	—	1	—	3	5	3	6	7	1	3	—	—	—	—	—	—	30
69—71	—	—	—	—	—	—	2	2	5	4	4	4	4	—	—	—	—	—	25
72—74	—	—	—	—	1	2	2	3	4	5	—	1	1	—	1	1	—	—	21
75—77	—	—	—	—	—	—	1	2	2	—	2	—	—	—	—	—	—	—	7
78—80	—	—	—	—	—	—	1	2	2	—	—	—	—	—	—	—	—	—	5
81—83	—	—	—	—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	2
84—86	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	2
Totals	1	2	2	3	22	34	72	98	125	153	140	132	73	41	18	7	3	1	927

TABLE XXXV.

Mentality.	96·5	96·7	96·9	97·1	97·3	97·5	97·7	97·9	98·1	98·3	98·5	98·7	98·9	99·1	99·3	99·5	99·7	99·9	Totals
Weak-Minded	—	—	—	—	1	4	10	12	24	25	34	31	24	15	8	2	1	—	191
Normal ...	1	2	2	3	21	30	62	86	101	128	106	101	49	26	10	5	2	1	736
Totals	1	2	2	3	22	34	72	98	125	153	140	132	73	41	18	7	3	1	927

TABLE XXXVI.

Labour.	96·5	96·7	96·9	97·1	97·3	97·5	97·7	97·9	98·1	98·3	98·5	98·7	98·9	99·1	99·3	99·5	99·7	99·9	Totals
Hard ...	—	1	2	1	15	19	36	48	66	88	94	77	37	27	13	5	3	1	533
Light ...	1	1	—	2	7	14	36	48	59	62	44	53	34	13	4	2	—	—	380
Totals	1	2	2	3	22	33	72	96	125	150	138	130	71	40	17	7	3	1	913

TABLE XXXVII.

Normal-minded only. Temperature. Age.

	21—	24—	27—	30—	33—	36—	39—	42—	45—	48—	51—	54—	57—	60—	63—	66—	69—	72—	75—	78—	81—	84—	Totals
96·5—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
96·7—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	2
96·9—	—	—	1	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	2
97·1—	—	1	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	3
97·3—	—	1	5	2	2	3	—	2	—	—	—	4	—	—	1	—	—	1	—	—	—	—	21
97·5—	—	1	4	5	4	—	2	—	1	—	4	—	—	3	1	3	—	2	—	—	—	—	30
97·7—	1	3	4	4	5	6	2	6	1	4	4	—	5	4	1	5	2	2	1	1	1	—	62
97·9—	—	3	9	5	5	9	8	7	3	4	2	6	—	5	7	3	2	3	2	2	—	1	86
98·1—	1	3	8	14	5	10	5	4	7	2	1	6	5	5	6	6	5	4	2	2	—	—	101
98·3—	—	5	10	15	9	12	10	12	5	4	3	3	5	6	12	7	4	5	—	—	—	1	128
98·5—	3	6	9	20	11	11	11	5	4	1	1	1	4	7	5	1	4	—	2	—	—	—	106
98·7—	3	5	11	14	10	12	8	5	5	3	1	—	4	3	8	3	4	1	—	—	1	—	101
98·9—	1	2	2	5	4	4	7	3	2	2	2	1	1	5	3	—	4	1	—	—	—	—	49
99·1—	2	2	4	4	3	5	2	1	—	1	—	—	—	2	—	—	—	—	—	—	—	—	26
99·3—	—	—	—	2	1	1	1	—	1	—	—	2	1	—	—	—	—	1	—	—	—	—	10
99·5—	1	—	—	—	1	—	1	1	—	—	—	—	—	—	—	—	—	1	—	—	—	—	5
99·7—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
99·9—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
Totals	12	32	68	90	61	73	58	46	29	21	19	24	25	41	45	30	25	21	7	5	2	2	736

TABLE XXXVIII.

Weak-minded only. Temperature. Age.

	21—	24—	27—	30—	33—	36—	39—	42—	45—	48—	51—	54—	57—	60—	63—	Totals
97·3—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
97·5—	—	2	—	—	—	—	—	—	—	1	1	—	—	—	—	4
97·7—	—	3	—	—	—	—	3	—	—	1	1	1	—	—	1	10
97·9—	—	2	—	3	2	—	1	—	2	—	2	—	—	—	—	12
98·1—	1	3	2	1	4	—	2	4	1	3	1	—	—	—	2	24
98·3—	—	4	4	—	3	1	2	—	2	3	2	4	—	—	—	25
98·5—	—	5	6	3	10	1	3	3	—	2	—	—	—	—	1	34
98·7—	1	3	3	3	4	2	1	4	3	1	3	2	—	—	1	31
98·9—	2	3	1	2	3	3	5	2	1	—	1	1	—	—	—	24
99·1—	—	2	1	3	1	—	4	1	—	1	—	1	—	—	1	15
99·3—	—	3	—	2	—	1	—	1	—	—	—	1	—	—	—	8
99·5—	—	—	—	—	1	—	—	—	—	—	—	—	1	—	—	2
99·7—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	5	31	17	17	28	8	21	15	9	12	11	10	1	—	6	191

TABLE XXXIX.

Temperature.

Totals	5	31	17	17	28	8	21	15	9	12	11	10	1	—	6	191

TABLE XXXIX.

Temperature.

Pulse.	96·5	96·7	96·9	97·1	97·3	97·5	97·7	97·9	98·1	98·3	98·5	98·7	98·9	99·1	99·3	99·5	99·7	99·9	Totals
45—48	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	2
49—52	—	—	—	—	1	—	1	—	3	—	—	—	—	—	—	—	—	—	5
53—56	—	—	—	—	—	2	3	2	4	4	1	1	—	—	—	—	—	—	17
57—60	1	—	1	—	2	3	7	6	9	13	6	3	2	1	3	—	—	—	57
61—64	—	1	—	—	1	7	9	14	9	21	14	6	6	2	—	—	—	—	90
65—68	—	—	—	2	5	10	11	19	19	28	21	23	9	1	1	1	—	—	150
69—72	—	—	—	—	4	2	7	20	17	21	20	20	5	3	1	—	—	—	120
73—76	—	—	1	1	3	6	12	11	18	14	18	23	14	9	1	1	—	—	132
77—80	—	—	—	—	4	2	9	9	20	16	16	13	8	7	2	1	2	—	109
81—84	—	1	—	—	—	2	3	9	12	12	13	10	14	6	2	2	—	—	86
85—88	—	—	—	—	—	—	2	3	7	13	12	12	5	3	3	1	1	1	63
89—92	—	—	—	—	—	—	4	2	6	3	10	8	4	5	—	—	—	—	42
93—96	—	—	—	—	1	—	2	1	1	3	1	1	1	1	2	1	—	—	15
97—100	—	—	—	—	—	—	—	1	—	2	6	3	3	1	2	—	—	—	18
101—104	—	—	—	—	—	—	—	—	—	2	2	2	1	1	1	—	—	—	9
105—108	—	—	—	—	—	—	—	1	—	—	—	3	1	—	—	—	—	—	5
109—112	—	—	—	—	—	—	—	—	—	1	—	2	—	—	—	—	—	—	3
113—116	—	—	—	—	—	—	—	—	—	—	—	2	—	1	—	—	—	—	3
Totals	1	2	2	3	21	34	72	98	125	153	140	132	73	41	18	7	3	1	926

TABLE XL.

Weight in lbs.

Mentality.	90—	100—	110—	120—	130—	140—	150—	160—	170—	180—	190—	200—	210—	220—	Totals
Normal ...	—	3	18	51	94	106	58	35	21	4	3	1	—	2	396
Weak-Minded	1	1	13	18	34	15	10	5	—	—	—	—	—	—	97
Totals	1	4	31	69	128	121	68	40	21	4	3	1	—	2	493

TABLE XLI.

Age.

Mentality.	21—	24—	27—	30—	33—	36—	39—	42—	45—	48—	51—	54—	57—	60—	63—	66—	69—	72—	75—	78—	81—	84—	Totals
Normal ...	7	18	37	48	34	41	32	26	16	12	11	13	13	21	23	15	13	11	4	3	1	1	400
Weak-Minded	3	16	9	9	14	4	11	8	5	6	6	5	1	—	3	—	—	—	—	—	—	—	100
Totals	10	34	46	57	48	45	43	34	21	18	17	18	14	21	26	15	13	11	4	3	1	1	500

TABLE XLII.

Temperature.

Normal-minded only. Pulse.	96·5	96·7	96·9	97·1	97·3	97·5	97·7	97·9	98·1	98·3	98·5	98·7	98·9	99·1	99·3	99·5	99·7	99·9	Totals
45—48	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	2
49—52	—	—	—	—	1	—	1	—	3	—	—	—	—	—	—	—	—	—	5
53—56	—	—	—	—	—	2	3	2	4	4	—	1	—	—	—	—	—	—	16
57—60	1	—	1	—	2	2	7	6	9	11	3	3	1	—	1	—	—	—	47
61—64	—	1	—	—	1	7	9	13	6	16	10	6	4	1	—	—	—	—	74
65—68	—	—	—	2	5	9	9	18	16	26	16	20	7	1	1	1	—	—	131
69—72	—	—	—	—	4	2	5	18	14	16	16	18	3	3	—	—	—	—	99
73—76	—	—	1	1	3	4	12	10	14	12	13	16	11	5	1	—	—	—	103
77—80	—	—	—	—	3	2	6	5	16	13	15	11	5	3	2	1	2	—	84
81—84	—	1	—	—	—	2	3	8	9	11	9	5	10	4	2	2	—	—	66
85—88	—	—	—	—	—	—	1	3	4	11	10	9	3	2	1	1	—	1	46
89—92	—	—	—	—	—	—	2	1	5	2	7	5	3	5	—	—	—	—	30
93—96	—	—	—	—	1	—	2	1	1	2	—	1	1	—	1	—	—	—	10
97—100	—	—	—	—	—	—	—	1	—	1	5	2	1	—	—	—	—	—	10
101—104	—	—	—	—	—	—	—	—	—	2	2	—	—	1	1	—	—	—	6
105—108	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	2
109—112	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	2
113—116	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	2
Totals	1	2	2	3	20	30	62	86	101	128	106	101	49	26	10	5	2	1	735

TABLE XLIII.

Temperature.

Weak-minded only. Pulse.	97·3	97·5	97·7	97·9	98·1	98·3	98·5	98·7	98·9	99·1	99·3	99·5	99·7	Totals
53—56	—	—	—	—	—	—	1	—	—	—	—	—	—	1
57—60	—	1	—	—	—	2	3	—	1	1	2	—	—	10
61—64	—	—	—	1	3	5	4	—	2	1	—	—	—	16
65—68	—	1	2	1	3	2	5	3	2	—	—	—	—	19
69—72	—	—	2	2	3	5	4	2	2	—	1	—	—	21
73—76	—	2	—	1	4	2	5	7	3	4	—	1	—	29
77—80	1	—	3	4	4	3	1	2	3	4	—	—	—	25
81—84	—	—	—	1	3	1	4	5	4	2	—	—	—	20
85—88	—	—	1	—	3	2	2	3	2	1	2	—	1	17
89—92	—	—	2	1	1	1	3	3	1	—	—	—	—	12
93—96	—	—	—	—	—	1	1	—	—	1	1	1	—	5
97—100	—	—	—	—	—	1	1	1	2	1	2	—	—	8
101—104	—	—	—	—	—	—	—	2	1	—	—	—	—	3
105—108	—	—	—	1	—	—	—	1	1	—	—	—	—	3
109—112	—	—	—	—	—	—	—	1	—	—	—	—	—	1
113—116	—	—	—	—	—	—	—	1	—	—	—	—	—	1
Totals	1	4	10	12	24	25	34	31	24	15	8	2	1	191

TABLE XLIV.

External Temperature.

Totals	1	4	10	12	24	25	34	31	24	15	8	2	1	191

TABLE XLIV.

External Temperature.

Temperature.	*40—*	*42—*	*44—*	*46—*	*48—*	*50—*	*52—*	*54—*	*56—*	*58—*	*60—*	*62—*	*64—*	*66—*	*68—*	*70—*	*72—*	*74—*	*76—*	*78—*	Totals
96·5—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
96·7—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
96·9—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	2
97·1—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	1	—	—	—	—	—	3
97·3—	—	—	1	—	—	1	1	1	—	3	4	1	1	—	2	—	1	2	—	1	19
97·5—	—	—	4	1	—	—	1	4	1	1	3	1	3	1	2	1	5	—	—	1	29
97·7—	—	—	3	3	—	1	1	3	3	5	10	1	8	4	5	2	8	4	—	1	62
97·9—	—	—	8	1	—	4	1	3	3	12	14	7	6	5	4	4	11	7	—	2	92
98·1—	1	—	8	4	—	4	4	1	4	17	21	6	9	9	8	3	6	9	2	1	117
98·3—	3	—	9	5	—	6	4	4	5	16	14	11	11	8	11	3	18	9	2	2	141
98·5—	2	—	11	4	—	6	5	5	5	11	16	16	16	2	6	4	11	4	—	1	125
98·7—	2	—	5	3	—	5	7	4	8	11	13	11	12	12	9	3	14	1	2	1	123
98·9—	—	—	2	—	—	—	4	1	4	7	5	6	8	3	11	1	9	2	1	—	64
99·1—	—	—	2	—	—	—	2	—	2	5	4	5	2	1	3	—	3	1	2	—	32
99·3—	—	—	—	—	—	—	—	—	2	1	5	2	3	—	1	—	—	1	—	—	15
99·5—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	1	1	1	—	—	—	5
99·7—	—	—	—	—	—	—	—	—	1	1	—	—	1	—	—	—	—	—	—	—	3
99·9—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
Totals	8	—	55	21	—	29	30	27	39	90	109	67	80	46	65	22	88	41	9	10	836

TABLE XLV.

Pulse.																					
45—48	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	—	—	—	2
49—52	—	—	—	—	—	1	1	—	—	1	—	—	—	—	1	1	—	—	—	—	5
53—56	—	—	—	1	—	1	1	—	1	—	3	1	2	—	2	—	3	—	—	1	16
57—60	1	—	3	—	—	3	1	6	1	5	14	3	3	2	5	—	3	2	—	—	52
61—64	2	—	4	2	—	5	2	5	1	11	10	4	5	6	8	1	9	3	1	—	79
65—68	—	—	10	4	—	2	5	6	8	19	17	10	11	7	5	2	20	3	1	2	132
69—72	3	—	10	3	—	4	5	1	3	10	16	12	11	7	6	4	8	4	—	3	110
73—76	1	—	9	2	—	6	5	3	7	15	17	5	12	9	9	2	12	5	1	1	121
77—80	—	—	6	4	—	1	7	2	5	9	9	11	12	3	8	1	9	5	4	2	98
81—84	—	—	5	2	—	3	2	1	4	12	9	10	6	4	4	4	6	6	—	—	78
85—88	—	—	4	1	—	2	—	2	3	3	8	4	6	4	5	2	7	7	—	—	58
89—92	1	—	1	2	—	—	1	1	1	3	3	2	4	1	5	4	5	4	1	—	39
93—96	—	—	1	—	—	—	—	—	1	—	2	—	2	—	1	—	3	—	—	1	11
97—100	—	—	1	—	—	1	—	—	—	1	—	3	3	2	3	—	2	—	1	—	17
101—104	—	—	1	—	—	—	—	—	1	—	—	1	1	—	1	1	1	1	—	—	8
105—108	—	—	—	—	—	—	—	—	1	1	1	—	—	—	2	—	—	—	—	—	5
109—112	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	2
113—116	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	—	—	—	2
Totals	8	—	55	21	—	29	30	27	39	90	109	66	80	46	65	22	88	41	9	10	835

TABLE XLVI. Respirations per minute.

External Temperature.	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	Totals
40—	—	—	—	—	—	—	2	—	—	1	3	1	—	—	—	—	—	—	1	—	—	—	8
42—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
44—	—	—	—	3	2	4	3	5	5	6	3	9	7	4	2	2	—	—	—	—	—	—	55
46—	—	—	—	1	—	1	3	2	5	1	1	—	3	1	1	1	—	—	—	—	1	—	21
48—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
50—	—	—	—	1	2	1	3	3	5	4	3	4	—	1	—	1	—	—	1	—	—	—	29
52—	—	—	—	—	—	4	3	3	3	3	4	4	3	—	1	1	—	—	—	—	—	1	30
54—	—	—	2	—	3	3	4	2	4	2	3	3	—	—	—	—	—	1	—	—	—	—	27
56—	—	—	—	2	2	4	4	3	7	3	3	4	2	2	2	1	—	—	—	—	—	—	39
58—	—	—	—	3	7	8	8	8	17	12	5	12	6	2	1	1	—	—	—	—	—	—	90
60—	1	—	2	1	5	8	12	11	11	20	7	10	10	7	2	1	—	—	—	1	—	—	109
62—	—	—	—	—	4	2	10	8	10	6	2	3	9	4	1	3	2	2	—	—	1	—	67
64—	—	—	—	2	5	6	3	11	7	11	7	9	5	6	5	—	—	1	—	—	—	—	78
66—	—	1	—	1	3	2	1	4	8	2	11	3	4	2	1	2	—	—	—	—	1	—	46
68—	1	—	—	—	2	6	2	6	8	8	4	10	3	2	3	2	—	3	3	2	—	—	65
70—	—	—	—	—	1	—	—	1	3	3	2	1	4	1	—	3	2	—	—	1	—	—	22
72—	—	1	1	—	4	6	11	8	5	12	9	6	8	10	3	3	1	—	—	—	—	—	88
74—	—	—	1	1	2	—	4	—	5	4	5	2	4	5	2	2	1	3	—	—	—	—	41
76—	—	—	—	—	—	—	2	1	2	—	3	—	—	1	—	—	—	—	—	—	—	—	9
78—	—	1	—	—	—	1	1	1	1	2	2	—	—	1	—	—	—	—	—	—	—	—	10
Totals	2	3	6	15	42	56	76	77	106	100	77	81	68	49	24	23	6	10	5	4	3	1	834

TABLE XLVII. Muscularity.

Nutrition.	Musc.	Weak	Totals
Thin	206	77	283
Fat	173	37	210
Totals	379	114	493

TABLE XLVIII. Prison Labour.

	Hard	Light	Totals
Thin	140	142	282
Fat	153	57	210
Totals	293	199	492

TABLE XLIX. General Health.

Nutrition.	Good	Poor	Totals
Thin	161	122	283
Fat	174	36	210
Totals	335	158	493

ALPHABETICAL INDEX TO CORRELATION TABLES IN APPENDIX.

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VARIATION AND CORRELATION OF THE NUMBER OF UMBEL RAYS OF SOME UMBELLIFERAE*.

By C. COOL and A. N. KOOPMANS,

Laboratory of Microscopical Anatomy, Technical High School, Delft.

(1) Since the investigations by Ludwig† into the variation of the number of umbel rays of the umbels of Umbelliferae, but little is to be found about this subject in biological literature, although the reading of the chapter "Summationskurven der Umbelliferen" of the article above-mentioned might suggest various problems.

For example, in the variation curves of Ludwig the numbers of the Fibonnacci series do not exclusively appear as maxima, but in some cases also the numbers of the first parallel series (1, 3, 4, 7, 11, etc.) appear. Vogler‡ also finds in his investigations into *Astrantia major* that principal series as well as parallel series occur in this case.

The few investigations by Ludwig on the Umbelliferae are hardly sufficient to allow us to state that modes which arise in the numbers of the first parallel series of these variation curves are with any degree of certainty only apparent modes, or to assert that they have resulted from the mixing of plants of different local races. On the contrary it is very probable that these numbers of the first parallel series are a characteristic phenomenon of the variation curves of the number of umbel rays of many species of Umbelliferae.

Of the many countings which we made in the summer of 1912 to find out something more about this subject, only those relating to three species of plants, *Pastinaca sativa*, *Anthriscus sylvestris* and *Sium latifolium*, are used here.

Only numbers of umbel rays of terminal umbels and those of lateral umbels of the first order were counted.

* Our thanks are due to Prof. Dr G. van Iterson who kindly revised this paper.

† *Bot. Centralblatt*, Bd. LXIV.

‡ *Beihefte zum botanischen Centralblatt*, Bd. XXIV.

(2) COUNTINGS OF *PASTINACA SATIVA*.

Variation of the number of umbel rays of the terminal umbel.

In Table I are arranged, according to the increasing numbers of umbel rays, terminal umbels of two groups of plants growing in different places; the first group of 341 plants came from clayey and peaty waysides in the neighbourhood of Delft (Holland), the second of 335 plants for the greater part from sheltered, for the smaller part from exposed spots in the duneland not far from the village of Lisse (Holland). The countings were made in the months of July and August of the year 1912.

In counting, a large number of fasciations were observed in the umbels. Among 515 plants, of which both the terminal and lateral umbels were counted, 167 were observed to have fasciations. As many rays were taken into account as there would have been, if no fasciation had occurred. Plate I, figures 1 and 2, shows some very interesting fasciations.

Variation of the number of umbel rays of the lateral umbels of the first order and correlation of the numbers of umbel rays of terminal umbels and that of lateral umbels.

The countings of umbel rays of lateral umbels have been combined in two different ways. In the first place lateral umbels belonging to plants with the same number of rays in the terminal umbel have been united into groups; and in the second place those which belonged to plants with the same number of lateral umbels.

In Table II, 3836 lateral umbels of 515 plants have been divided into groups, each of which has been placed in a horizontal row and contains all the lateral umbels of plants with the same number of rays in the terminal umbels.

The mean number of umbel rays of the lateral umbels is greater than that of the terminal umbels. For that reason terminal and lateral umbels are considered separately.

In Table III the lateral umbels are divided into groups in such a way that all the lateral umbels of plants with the same number of lateral umbels are arranged in one horizontal row.

The distribution of the number of umbel rays of the 3836 lateral umbels is represented graphically in Fig. 3.

The distribution, which is given in the first and the last column of Table III and which gives the number of lateral umbels per plant, has not been studied in detail because it is of no importance for this investigation. The constants $\beta_1 = 0{\cdot}144$ and $\beta_2 = 3{\cdot}485$, the "critical function" $= 0{\cdot}208$ and the curve which will fit best to this distribution is a Pearson curve of Type IV.

TABLE I.

Variation of the number of umbel rays of the terminal umbel of Pastinaca sativa.

Number of umbel rays of the terminal umbel	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	Totals
Number of terminal umbels, Delft	3	11	31	53	49	**55**	42	**53**	23	10	4	2	1	3	—	1	—	—	—	—	—	341
Number of terminal umbels, Lisse	2	14	29	34	**59**	53	40	32	22	12	**13**	9	7	2	3	2	—	—	1	—	1	335
Number of terminal umbels, Totals	5	25	60	87	**108**	**108**	82	**85**	45	22	17	11	8	5	3	3	—	—	1	—	1	676

Mean of umbel rays for 676 terminal umbels = 11·325.
S.D. of umbel rays for 676 terminal umbels = 2·746.

TABLE II.

Correlation of the number of umbel rays of terminal umbels and lateral umbels of Pastinaca sativa.

Number of umbel rays of the terminal umbel.	Number of lateral umbels with umbel rays																								Total number of lateral umbels	Number of plants
	5	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*		
6	—	1	3	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6	2
7	—	4	4	26	27	**38**	14	6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	119	21
8	2	5	13	39	46	**58**	49	20	20	4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	256	43
9	—	9	10	36	57	**76**	73	55	32	7	—	1	—	—	—	—	—	—	—	—	—	—	—	—	356	56
10	1	—	5	36	50	109	**152**	110	101	34	10	4	2	—	—	—	—	—	—	—	—	—	—	—	614	89
11	—	—	3	16	43	85	**147**	136	111	55	27	10	2	5	—	—	—	—	—	—	—	—	—	—	640	87
12	—	—	—	3	19	65	79	94	**104**	59	34	17	9	7	1	—	—	—	—	—	—	—	—	—	491	65
13	—	1	2	**12**	10	26	49	64	**87**	79	**83**	38	19	11	—	4	—	—	—	—	—	—	—	—	485	59
14	—	—	—	—	1	4	14	27	44	**54**	49	**52**	21	6	4	—	—	—	—	—	—	—	—	—	276	30
15	—	—	—	1	2	**17**	11	14	21	**26**	**26**	20	18	12	6	1	—	—	—	—	—	—	—	—	175	20
16	—	—	—	—	—	—	1	3	5	9	**28**	18	20	**30**	19	7	1	1	—	—	—	—	—	—	142	15
17	—	—	—	—	—	—	—	2	4	9	3	12	13	17	12	4	6	4	3	1	—	—	—	—	90	9
18	—	—	—	—	—	—	2	3	3	5	7	8	5	10	8	12	7	3	5	3	—	—	—	—	81	7
19	—	—	—	—	—	—	—	—	—	1	2	8	4	10	5	4	2	3	1	1	—	—	—	—	41	4
20	—	—	—	—	—	—	—	—	1	4	4	5	6	6	1	—	—	—	—	—	—	—	—	—	27	3
21	—	—	—	—	—	—	—	—	3	2	2	2	1	—	1	5	2	4	2	—	—	—	—	—	24	3
22	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
23	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
24	—	—	—	—	—	—	—	—	1	—	—	2	—	1	1	—	—	—	—	—	—	—	—	—	5	1
25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
26	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	4	—	2	1	8	1
Totals	3	20	40	171	255	478	591	534	537	348	275	197	120	115	58	37	18	15	11	6	4	—	2	1	3836	515

Mean of umbel rays for 515 lateral umbels = 12·549.
S.D. of umbel rays for 515 lateral umbels = 3·007.
Coefficient of correlation = 0·7269.

TABLE III.

Correlation of the number of lateral umbels per plant and the number of umbel rays of lateral umbels of Pastinaca sativa.

Number of lateral umbels per plant.	Number of lateral umbels with umbel rays																								Total number of lateral umbels	Number of plants
	5	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*		
2	—	—	1	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4	2
3	—	—	3	11	8	4	3	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	30	10
4	—	6	9	17	24	22	22	7	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	108	27
5	—	5	6	27	41	**80**	49	24	14	5	—	4	—	1	1	—	—	—	—	—	—	—	—	—	257	52
6	—	6	9	38	65	**102**	**102**	60	42	14	4	—	—	—	—	—	—	—	—	—	—	—	—	—	442	74
7	2	2	8	45	64	109	**127**	108	90	49	43	23	12	7	1	—	—	—	—	—	—	—	—	—	690	99
8	1	—	2	15	33	79	106	155	**160**	88	65	42	17	13	8	1	—	—	—	1	4	—	2	1	793	100
9	—	—	—	8	11	53	102	92	**120**	79	60	37	33	31	8	**12**	2	4	2	—	—	—	—	—	654	73
10	—	1	2	7	—	8	36	43	**59**	**60**	37	27	17	**30**	12	4	3	2	—	—	—	—	—	—	348	35
11	—	—	—	1	5	14	**24**	22	**25**	11	**26**	18	18	**22**	17	7	3	4	1	1	—	—	—	—	219	20
12	—	—	—	—	2	6	**17**	14	15	**31**	25	**31**	14	3	2	**7**	1	—	—	—	—	—	—	—	168	14
13	—	—	—	—	—	—	1	2	1	—	5	4	2	2	4	5	9	5	8	4	—	—	—	—	52	4
14	—	—	—	—	1	1	2	2	3	2	7	6	3	—	—	—	—	—	—	—	—	—	—	—	27	2
15	—	—	—	—	—	—	—	4	7	9	3	5	4	6	5	1	—	—	—	—	—	—	—	—	44	3
Totals	3	20	40	171	255	478	**591**	534	**537**	348	275	197	120	115	58	37	18	15	11	6	4	—	2	1	3836	515

If we consider the different distributions in the horizontal rows of Tables II and III in which the maximum numbers are printed in heavy figures, we become convinced that besides the numbers of the principal series such as 8 and 13 (and multiples of numbers of this series such as 15 and 16), also numbers of the first parallel series such as 11 and 18 (and perhaps multiples of numbers of that series, 14 for example) are not infrequent.

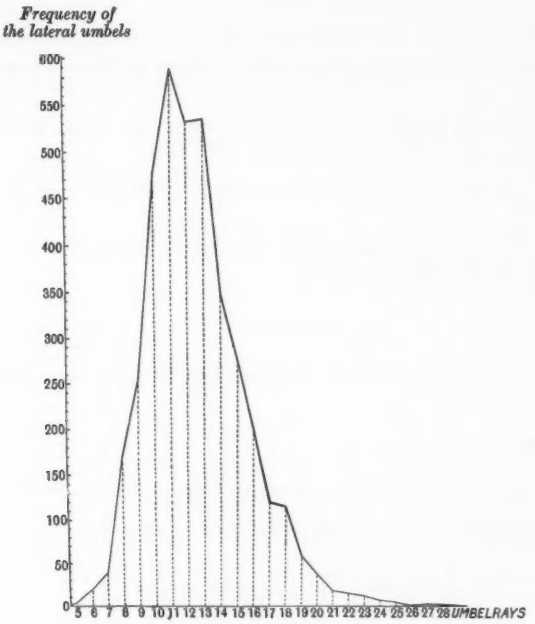


FIG. 3. *Pastinaca sativa.*

It is certainly very remarkable that with the increasing number of rays of the terminal umbel or with the increasing number of lateral umbels per plant, the lateral umbels that occur as maxima in a horizontal distribution possess a number of umbel rays that leaps up from 11 in one to 13 in another horizontal distribution; in other words:

The number of umbel rays 12 does not occur as a maximum in any of the horizontal groups of the two correlation tables.

(3) Countings of *Anthriscus sylvestris*.

Variation of the number of umbel rays of the terminal umbel.

A little preliminary investigation showed that the number of umbel rays of the terminal umbel of *Anthriscus sylvestris* is very small and that in most cases this umbel consists of 1, 2, 3 or 4 umbel rays and in some cases of 5, or is entirely missing.

Afterwards four countings were made:

(*a*) On 502 plants in the neighbourhood of Beverwijk on May 16th, 1912.

(*b*) On 305 plants in the neighbourhood of Lisse on May 26th and July 16th, 1912.

(*c*) On 77 plants partly in the neighbourhood of Lisse, partly in that of Bentveld, on July 6th, July 19th and July 27th, 1912.

(*d*) On 46 plants from the Schapenweg in the neighbourhood of Rijswijk on May 19th, 1912.

The result of these countings of the number of umbel rays of 930 terminal umbels is given in the following table.

TABLE IV.

Variation of the number of umbel rays of the terminal umbel of Anthriscus sylvestris.

Number of umbel rays of the terminal umbel	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	Totals
Number of terminal umbels. Group (*a*)	9	107	**196**	143	40	4	1	1	—	1	—	502
Number of terminal umbels. Group (*b*)	16	107	**112**	49	14	3	—	3	—	—	1	305
Number of terminal umbels. Group (*c*)	1	**26**	23	23	1	—	2	1	—	—	—	77
Number of terminal umbels. Group (*d*)	—	13	**15**	13	4	1	—	—	—	—	—	46
Number of terminal umbels. Totals	26	253	**346**	228	59	8	3	5	—	1	1	930

Mean of umbel rays for 930 terminal umbels = 2·125.
S.D. of umbel rays for 930 terminal umbels = 1·119.

The graphical representation in Fig. 4 of the distribution in the last row of Table IV is an asymmetrical polygon with mode at 2 umbel rays. The point which is obtained by taking the terminal umbels with 0 umbel rays into account

accords with the one that we can expect when we know only the other points and causes the variation polygon to become rather symmetrical with regard to the vertical through the mode.

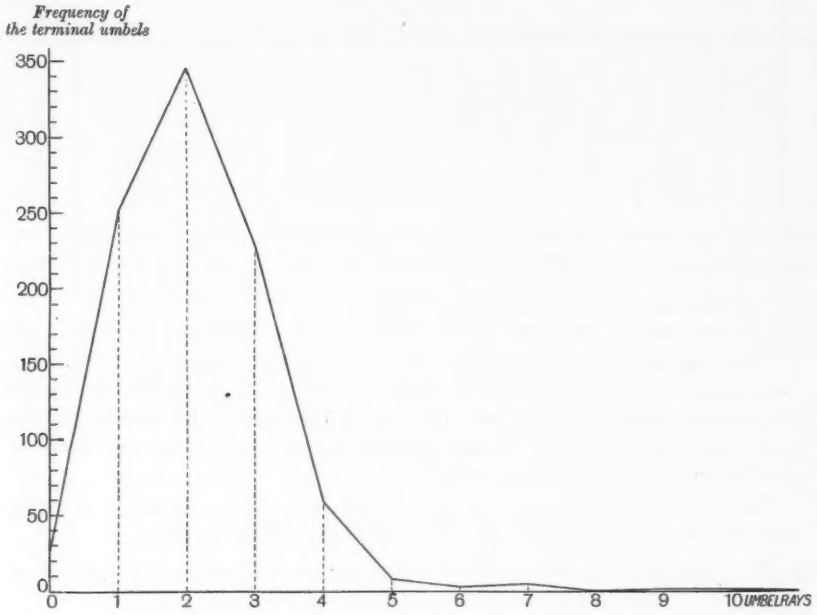


FIG. 4. *Anthriscus sylvestris.*

Variation of the number of umbel rays of the lateral umbels of the first order and correlation of the number of umbel rays of terminal umbels and that of lateral umbels.

Of 235 plants (being the same series for which the rays of the terminal umbels were counted and which were included in the preceding distributions) the number of umbel rays of the lateral umbels was determined.

Again these lateral umbels were divided into groups, in the first place into groups belonging to plants with the same number of rays in the terminal umbel, and in the second place into groups belonging to plants with the same number of lateral umbels.

In the following table have been assembled the results of the countings on 1632 lateral umbels of these 235 plants in which the first sub-division was taken. It is easy to see that the numbers 8 and 10 are predominant.

TABLE V. *Correlation of the number of umbel rays of terminal umbels and lateral umbels of* Anthriscus sylvestris.

Number of umbel rays of the terminal umbel.	Number of lateral umbels with 3 umbel rays	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	Total number of lateral umbels	Number of plants
0	—	2	6	5	11	19	13	13	2	—	—	—	—	—	—	—	—	71	9
1	2	23	27	37	55	**135**	119	107	44	17	1	—	—	—	—	—	—	567	77
2	3	5	15	22	43	**108**	97	**127**	73	49	15	5	—	—	—	—	—	562	82
3	1	3	6	7	19	30	45	**82**	68	30	24	7	2	3	—	—	—	327	50
4	—	1	—	—	4	2	3	8	17	7	4	3	1	—	—	—	—	50	9
5	1	1	—	—	2	5	—	2	5	1	1	2	1	—	1	—	—	22	3
6	—	1	—	3	2	3	1	1	1	—	1	—	—	—	—	—	—	13	2
7	—	—	1	1	1	2	1	4	2	1	1	1	1	1	1	1	1	20	3
Totals	7	36	55	75	137	**304**	279	**344**	212	105	47	18	5	4	2	1	1	1632	235

Mean of umbel rays for 235 lateral umbels = 9·127.
S.D. of umbel rays for 235 lateral umbels = 2·150. Coefficient of correlation = 0·3033.

The mean number of umbel rays of the lateral umbels is much greater than the mean number of rays of the terminal umbels. For that reason terminal and lateral umbels were considered separately. This great difference of the average numbers to a certain degree accounts for the fact that the coefficient of correlation is low, much lower than with *Pastinaca sativa*.

The second sub-division of the lateral umbels in groups belonging to plants with the same number of lateral umbels gives the following result:

TABLE VI. *Correlation of the number of lateral umbels per plant and the number of umbel rays of lateral umbels of* Anthriscus sylvestris.

Number of lateral umbels per plant.	Number of lateral umbels with 3 umbel rays	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	Total number of lateral umbels	Number of plants
2	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	2	1
3	—	—	—	—	2	1	—	—	—	—	—	—	—	—	—	—	—	3	1
4	—	—	—	—	—	—	—	1	3	2	—	2	—	—	—	—	—	8	2
5	—	—	1	2	8	19	22	**24**	23	6	5	—	1	1	1	1	1	115	23
6	—	3	5	6	28	56	61	**74**	48	34	18	6	2	—	1	—	—	342	57
7	1	11	22	27	41	**125**	98	**119**	69	30	8	1	—	—	—	—	—	552	79
8	4	10	12	23	36	**55**	54	**75**	47	22	15	9	2	3	—	—	—	367	46
9	—	8	11	10	15	32	36	**43**	17	7	1	—	—	—	—	—	—	180	20
10	2	3	2	4	6	**15**	7	8	3	—	—	—	—	—	—	—	—	50	5
11	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
13	—	1	2	1	1	1	1	—	2	4	—	—	—	—	—	—	—	13	1
Totals	7	36	55	75	137	**304**	279	**344**	212	105	47	18	5	4	2	1	1	1632	235

In this table also the numbers 8 and 10 are conspicuous just as in the subdivisions not reproduced here of each horizontal row in the above table, which were made according to the number of rays of the terminal umbel.

It is remarkable that there is not a single case of *Anthriscus sylvestris* in which a number of 9 umbel rays in a horizontal row of one of the Tables V and VI occurs as a maximum, whereas the numbers 8 and 10 not infrequently occur as modes.

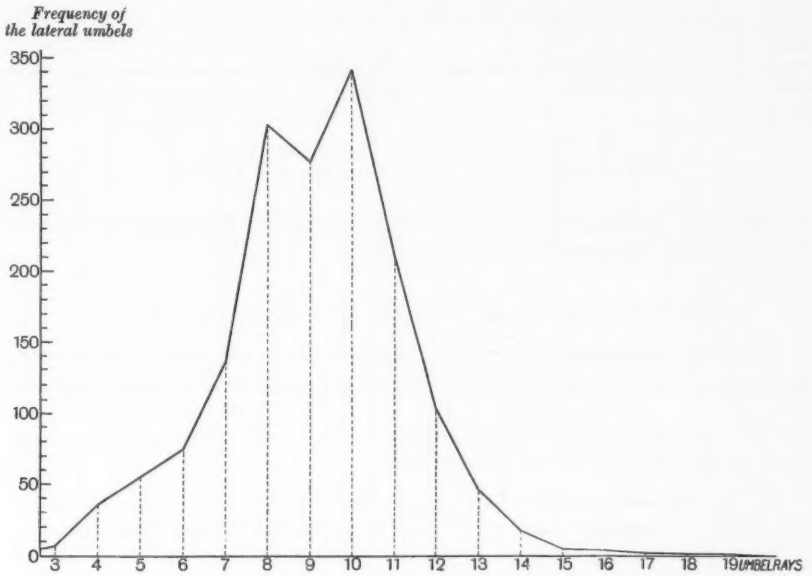


Fig. 5. *Anthriscus sylvestris.*

Figure 5 gives a graphical representation of the variability of the number of umbel rays of these 1632 lateral umbels.

(4) Countings of *Sium latifolium.*

Variation of the number of umbel rays of the terminal umbel.

The number of plants examined of *Sium latifolium* was much smaller than the number of plants of the preceding species, which were submitted to examination. Only in the case of 127 plants were the umbel rays of terminal umbels and lateral umbels counted, and of one only of the umbel rays of the lateral umbels, because the terminal umbel was missing.

The distribution of the numbers of umbel rays of 127 terminal umbels was as follows:

TABLE VII.

Variation of the number of umbel rays of the terminal umbel of Sium latifolium.

Number of umbel rays of the terminal umbel	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*36*	*37*	Total
Number of terminal umbels	3	4	3	6	6	1	6	8	3	5	7	10	4	4	6	9	10	9	8	5	4	4	1	1	127

Mean of umbel rays for 127 terminal umbels = 25·638.
S.D. of umbel rays for 127 terminal umbels = 5·948.

Conclusions could not be drawn from this distribution, the number of variates being too small with respect to the great range of variation.

Variation of the number of umbel rays of the lateral umbels.

The lateral umbels again were sub-divided into groups belonging to plants with the same number of rays in the terminal umbels and into groups belonging to plants with the same number of lateral umbels. As the former groups were very small, they can give little or no information about preference for certain numbers. The latter groups have been united in Table VIII.

Though the small number of umbels, of which the rays were counted, and the great range of variability make it necessary to be prudent in drawing conclusions, we think it probable that not only the numbers of the principal series are of importance in this case, but also other numbers as for instance 18 and 29.

Correlations.

The coefficient of correlation for the number of rays of terminal and lateral umbels was as follows:

For *Pastinaca sativa* ... ... 0·7269 ± 0·0051.
„ *Anthriscus sylvestris* ... 0·3033 ± 0·0152.
„ *Sium latifolium* ... ... 0·8693 ± 0·0070.

It is remarkable that this coefficient of correlation is different for these three species of plants, and it would be worth examining if this coefficient has a constant value within certain limits for different local races of the same species of plants. In that case a number would have been found, characteristic of each particular species.

TABLE VIII.

Correlation of the number of lateral umbels per plant and the number of umbel rays of lateral umbels of Sium latifolium.

Number of lateral umbels per plant.	Number of lateral umbels with umbel rays																														Total number of lateral umbels	Number of plants
	13	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*36*	*37*	*38*	*39*	*40*	*41*	*42*		
2	1	4	0	1	0	0	1	2	0	1	0	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	11	6
3	2	1	0	4	2	3	1	4	5	3	3	3	4	2	1	2	6	4	2	1	—	—	—	—	—	—	—	—	—	—	53	19
4	—	—	—	3	3	8	6	3	10	11	7	4	9	5	7	3	2	1	6	3	5	3	1	2	1	—	—	1	—	—	104	27
5	—	—	—	—	4	2	0	3	5	4	9	11	6	4	9	6	7	4	12	15	6	8	12	8	5	3	3	1	1	—	148	33
6	—	—	—	1	0	1	0	2	1	2	2	5	4	6	5	5	9	3	5	14	19	21	16	10	12	13	10	7	4	—	177	32
7	—	—	—	—	—	—	—	2	3	—	—	—	1	1	—	5	0	0	2	2	6	3	5	8	13	4	4	8	1	1	69	11
Totals	3	5	0	9	9	14	8	16	24	21	21	24	24	18	22	21	24	12	27	35	36	35	34	28	31	20	17	17	6	1	562	128

Mean of umbel rays for 562 lateral umbels = 29·416.
S.D. of umbel rays for 562 lateral umbels = 6·782.
Coefficient of correlation = 0·8693.

Conclusions.

The result of this investigation is that even in relatively small groups of sufficiently homogeneous material of *Pastinaca sativa* numbers (or multiples of numbers) of the principal series, or numbers (or multiples of numbers) of the first parallel series, or both, occur as maxima in distributions of the number of rays of terminal and of lateral umbels.

The maximum numbers of umbel rays that occur in the horizontal rows of the correlation tables do not shift regularly from top to bottom, and in the case of *Pastinaca sativa* the number of 12 rays does not occur as a maximum in the groups of which the plants have the same number of rays in the terminal umbel or in those of which the plants have the same number of lateral umbels; on the contrary the numbers 11 and 13 occur as maxima.

It is therefore not probable that the numbers of the first principal series should be caused by the mixing of different local races of plants, in each of which numbers of the principal series occur as maxima, as Ludwig thinks is the case with other Umbelliferae. It is more probable that the numbers of the first parallel series must be considered as characteristic of the variation of many species of Umbelliferae.

ON THE PARTIAL CORRELATION-RATIO

PART II. NUMERICAL

Being Part II of a Thesis for the degree of D.Sc. in the University of London

By L. ISSERLIS, B.A.

1. The object of the present paper is to provide numerical illustrations of some of the results in the first part*. Suitable material is not easily obtained, as many tables dealing with three variable characters are artificially curtailed as regards one or more of them. I am indebted to Miss Ethel M. Elderton for having kindly placed at my disposal manuscript tables of the heights to the nearest inch and weights to the nearest pound of the boys in group A (comprising schools in the poorest parts of the city) of her investigations into the heights and weights of school children in Glasgow†.

I have arranged the material in tables which differ from Tables I–IX of Miss Elderton's paper in the following respects:

(i) A *uniform* grouping of 5 lb. intervals for weight and 3 inch intervals for height is adopted at all ages;

(ii) 384 boys of central age 5 (i.e. ages 4·5 to 5·5 years), omitted in Miss Elderton's tables, are here included, and two boys (age 6·5 to 7·5 years) whose height was below 21·5 inches, included in her tables, are omitted here.

These tables are cut off artificially by the regulations of the School Board with regard to school leaving age and the admission age of infants.

Tables I–X contain the data, the frequency distributions of height and weight being given for each of the ages from 5 to 14 of the 11,382 boys.

* "On the Partial Correlation Ratio," Part I, Theoretical, by L. Isserlis. *Biometrika*, Vol. x, 1914, pp. 391–411. (The title of the first part was badly chosen, the generalized H there introduced is a multiple one, providing a measure of the dependence of one variable on two or more other variables. An example of a partial η would be ${}_x\eta_{z \cdot y}$, being the correlation ratio of z on y for a given value of x. Pearson has recently published very simple formulae connecting the partial correlation ratio with the multiple correlation ratio: see *R. S. Proc.* Vol. 91, A, pp. 492–8. They suffice to show that a knowledge of the multiple correlation ratios leads at once to the partial correlation ratios, and to that extent justify my title.)

† "Height and Weight of School Children in Glasgow," by Ethel M. Elderton. *Biometrika*, Vol. x, 1914, pp. 288–339.

Tables XI, XII, XIII give the distributions of height and weight, height and age, and age and weight for the whole of the individuals in the preceding ten tables. They are summaries. The *direct* calculation of one of the three generalized H's, say that of age on height and weight, requires the use of Tables I–X, while the various approximations given in the paper cited can be obtained from the summaries in Tables XI–XIII.

I have calculated the correlation ratio of age on height and weight, and that of weight on age and height directly, and compared them with the values obtained as approximations from the summaries.

The direct calculation is laborious and the results show that the various approximation formulae are in general sufficiently accurate to enable us to dispense with this heavy labour.

Throughout the present paper, I denote the age by 'z' with an arbitrary origin at 10 years, the height by 'x' with the origin at 49 inches and the weight by 'y' with origin at 56 lbs.

2. With the arbitrary origin at 49 inches, the raw moments for the height frequencies are:

$$\begin{aligned} \bar{x} = p'_x &= -0{\cdot}511861\text{*}, \\ p'_{x^2} &= \ \ 3{\cdot}453699, \\ p'_{x^3} &= -4{\cdot}938324, \\ p'_{x^4} &= \ 30{\cdot}678791. \end{aligned}$$

Thus the mean height is $49 - 3\,(0{\cdot}511861) = 47{\cdot}46$ inches.

The moments referred to mean are given by the formulae:

$$\begin{aligned} p_{x^2} &= p'_{x^2} - (p'_x)^2 = 3{\cdot}191697, \\ p_{x^3} &= p'_{x^3} - 3p'_{x^2}p'_x + 2p'^3_x = 0{\cdot}096908, \\ p_{x^4} &= p'_{x^4} - 4p'_{x^3}p'_x + 6p'_{x^2}p'^2_x - 3p'^4_x = 25{\cdot}7912. \end{aligned}$$

From these

$$_x\beta_1 = \frac{p_{x^3}{}^2}{p_{x^2}{}^3} = {\cdot}0003,$$

$$_x\beta_2 = \frac{p_{x^4}}{p_{x^2}{}^2} = 2{\cdot}5318.$$

The standard deviation is $\sigma_x = 1{\cdot}78653$ in 3 inch units, or 5·36 inches.

For weights the arbitrary origin is at 56 lbs., and the raw moments are:

$$\begin{aligned} \bar{y} = p'_y &= -0{\cdot}205412, \\ p'_{y^2} &= \ \ 6{\cdot}772096, \\ p'_{y^3} &= \ \ 5{\cdot}979090, \\ p'_{y^4} &= 134{\cdot}637146. \end{aligned}$$

* For the sake of uniformity the constants in all *intermediate* results are given to six places of decimals.

So that the mean weight is $56 - 5\,(0{\cdot}205412) = 55{\cdot}97$ lbs., and the moments referred to the mean are:

$$p_{y^2} = 6{\cdot}729902,$$
$$p_{y^3} = 10{\cdot}134965,$$
$$p_{y^4} = 141{\cdot}258877,$$

giving $\quad {}_y\beta_1 = {\cdot}3370, \qquad {}_y\beta_2 = 3{\cdot}1189.$

The standard deviation is $\sigma_y = 2{\cdot}594205$ in 5 lb. units, or 12·97 lbs.

With arbitrary origin at 10 years, the raw moments of the ages are:

$$\bar{z} = p'_z = -\,0{\cdot}568617,$$
$$p'_{z^2} = 6{\cdot}402214,$$
$$p'_{z^3} = -\,9{\cdot}423124,$$
$$p'_{z^4} = 80{\cdot}916710.$$

The mean age is therefore $10 - 0{\cdot}568617 = 9{\cdot}43$ years, and the moments about the mean age are:

$$p_{z^2} = 6{\cdot}078889,$$
$$p_{z^3} = 1{\cdot}130401,$$
$$p_{z^4} = 71{\cdot}591474,$$

leading to $\quad {}_z\beta_1 = {\cdot}0057, \qquad {}_z\beta_2 = 1{\cdot}9374,$

while the standard deviation is $\sigma_z = 2{\cdot}47$ years.

3. The mixed moments p_{xy}, p_{xy^2}, etc., can be found from the correlation tables XI–XIII. The values of the raw moments required are as follows:

$$p'_{xy} = 4{\cdot}311017, \qquad p'_{xx} = 3{\cdot}949042,$$
$$p'_{xy^2} = -\,1{\cdot}066245, \qquad p'_{xx^2} = -\,7{\cdot}108065,$$
$$p'_{xy^3} = 74{\cdot}045335, \qquad p'_{xx^3} = 50{\cdot}313120,$$
$$p'_{x^2y} = -\,3{\cdot}712880, \qquad p'_{x^2z} = -\,5{\cdot}488842,$$
$$p'_{x^3y} = 35{\cdot}657353, \qquad p'_{x^3z} = 24{\cdot}225619,$$

$$p'_{xy} = 5{\cdot}430241, \quad p'_{x^2y^2} = 47{\cdot}927956, \quad p'_{xyz} = -\,4{\cdot}341768, \quad p'_{x^2z^2} = 37{\cdot}510631.$$

The corresponding moments about the mean are:

$$p_{xy} = p'_{xy} - \bar{x}\bar{y} = 4{\cdot}205875,$$
$$p_{yz} = 5{\cdot}313440,$$
$$p_{zx} = 3{\cdot}657989,$$
$$p_{xy^2} = p'_{xy^2} - 2\bar{y}p'_{xy} - \bar{x}p'_{y^2} + 2\bar{x}\bar{y}^2 = 4{\cdot}128088,$$
$$p_{xy^3} = p'_{xy^3} - 3\bar{y}p'_{xy^2} + 3\bar{y}^2p'_{xy} - \bar{x}p'_{y^3} + 3\bar{x}\bar{y}p'_{y^2} = 79{\cdot}144020,$$
$$p_{x^2y^2} = p'_{x^2y^2} - 2\bar{y}p'_{x^2y} - 2\bar{x}p'_{xy^2} + \bar{y}^2p'_{x^2} + \bar{x}^2p'_{y^2} + 4\bar{x}\bar{y}p'_{xy} - 3\bar{x}^2\bar{y}^2 = 49{\cdot}011115,$$
$$p_{xyz} = p'_{xyz} - \bar{x}\bar{y}\bar{z} - \bar{x}p_{yz} - \bar{y}p_{zx} - \bar{z}p_{xy} = 1{\cdot}580733.$$

Similar formulae give:

$$p_{x^2y} = 1{\cdot}302279, \qquad p_{x^3z} = {\cdot}219749,$$
$$p_{xy^3} = 33{\cdot}336761, \qquad p_{x^2z} = 19{\cdot}108627,$$
$$p_{xz^3} = {\cdot}328967, \qquad p_{xz^3} = 42{\cdot}785163.$$

4. Further we take: $q_{x^l y^m z^n} = \dfrac{p_{x^l y^m z^n}}{\sigma_{x^l}\sigma_{y^m}\sigma_{z^n}}$.

We now collect into a single table the values of the reduced moments, the correlation coefficients and the correlation ratios.

TABLE XIV.

q_{x^2y}	·157282	r_{xy}	·907489	—	—
q_{xy^2}	·343344	$_y\eta_x$	·918157	$_y\eta^2_x - r^2_{xy}$	·019478
$q_{x^2y^2}$	2·281729	$_x\eta_y$	·915430	$_x\eta^2_y - r^2_{xy}$	·014476
q_{x^3y}	2·253653	—	—	—	—
q_{xy^3}	2·537432	r_{yz}	·830729	—	—
q_{x^3}	·016995	$_y\eta_z$	·851196	$_y\eta^2_z - r^2_{zy}$	034424
q_{x^4}	2·531793	$_z\eta_y$	·833060	$_z\eta^2_y - r^2_{yz}$	·003878
q_{y^3}	·580509	—	—	—	—
q_{y^4}	3·118877	r_{zx}	·830463	—	—
q_{x^2z}	·027925	$_x\eta_z$	·844792	$_x\eta^2_z - r^2_{zx}$	·024005
q_{xz^2}	·030294	$_z\eta_x$	·831447	$_z\eta^2_x - r^2_{zx}$	·001635
q_{xz^3}	1·597889	—	—	—	—
q_{x^3z}	1·359207	—	—	—	—
$q_{x^2z^2}$	1·605274	—	—	—	—
q_{xyz}	·138335	—	—	—	—

It will be observed that the differences $_z\eta^2_y - r^2_{yz}$ and $_z\eta^2_x - r^2_{zx}$ are very small, showing that the regression lines of weight on age, and of height on age are nearly straight. The six empirical regression lines are given in Figs. 1–6.

FIG. 1. Regression of Age on Weight.

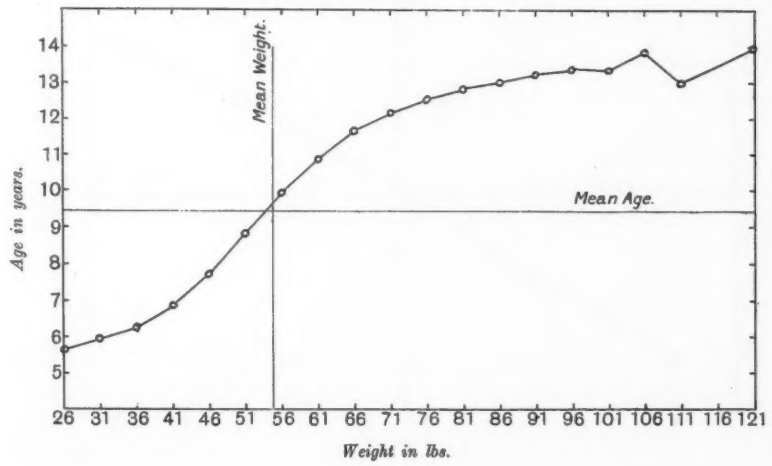


FIG. 2. Regression of Weight on Age.

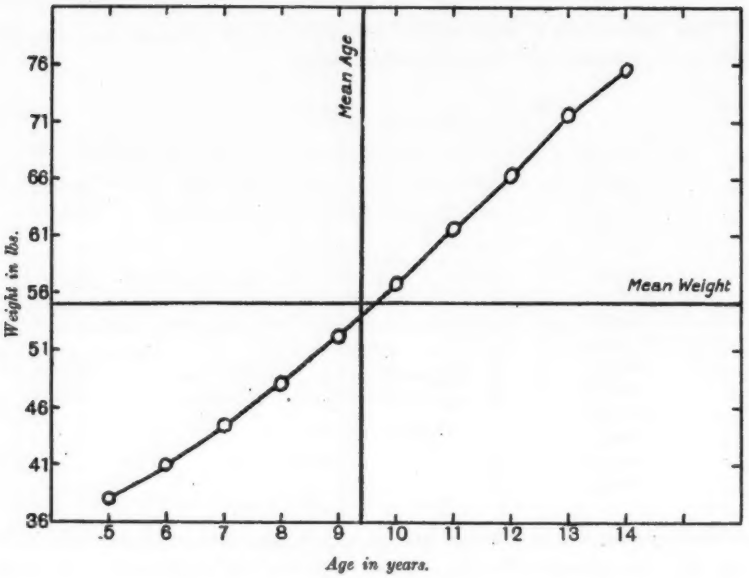


FIG. 3. Regression of Height on Age.

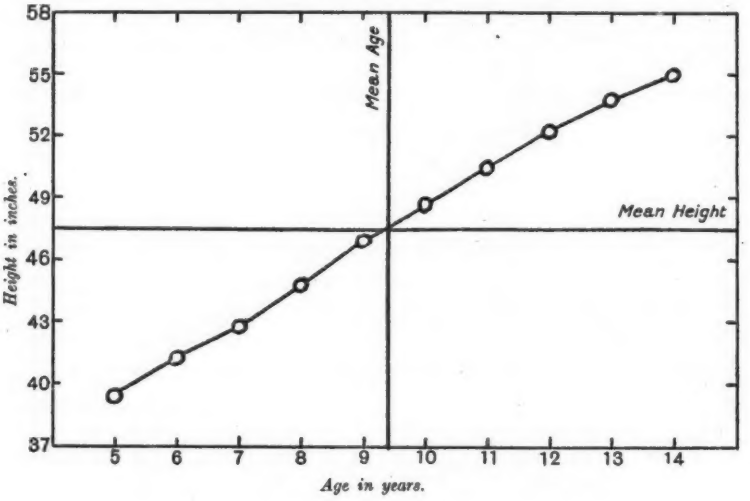


FIG. 4. Regression of Age on Height.

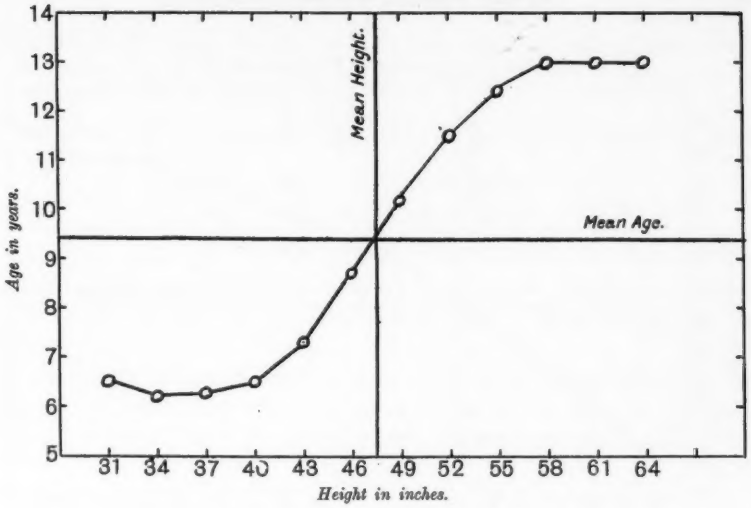


FIG. 5. Regression of Weight on Height.

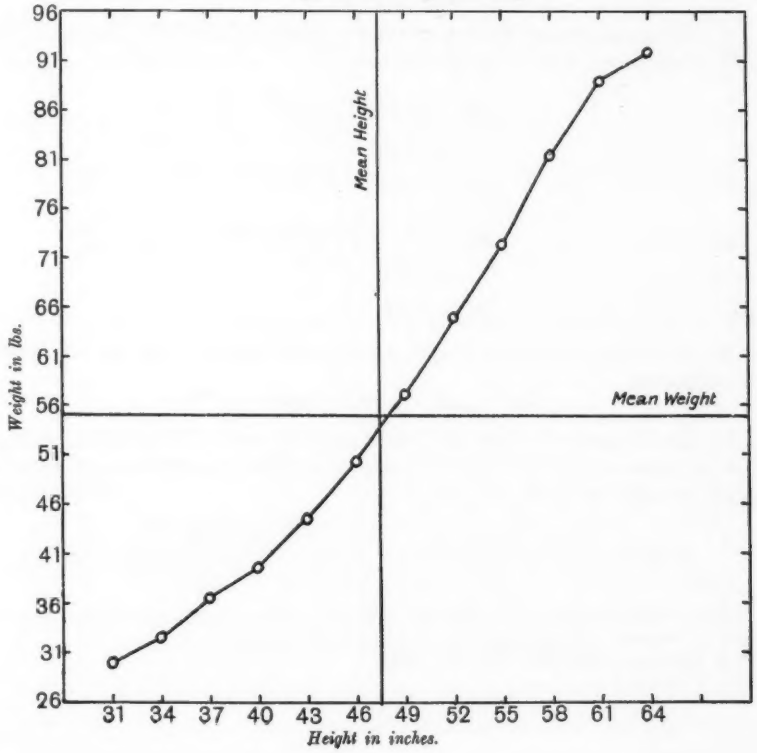
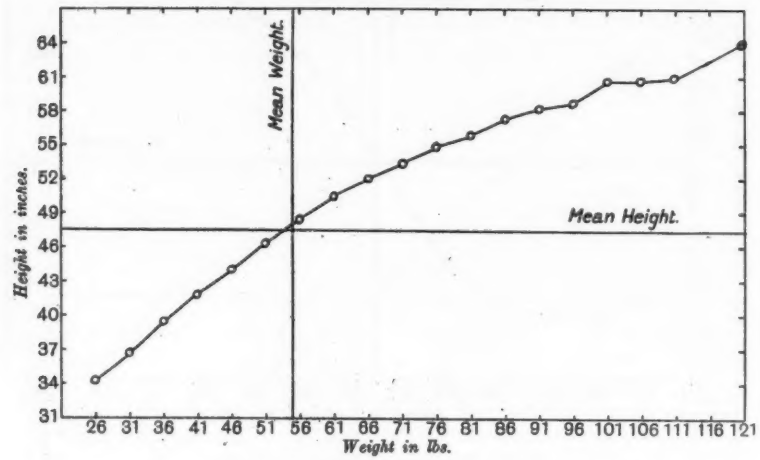


FIG. 6. Regression of Height on Weight.



5. The mean ages of the x, y arrays are given in Table XV. From this table ${}_{xy}H^2_z$ is calculated directly by the formula:

$$ {}_{xy}H^2_z = SS\frac{\{n_{xy}(\bar{z} - z_{xy})^2\}}{N\sigma_z^2} $$
$$ = \cdot 7519, $$

or
$$ {}_{xy}H_z = \cdot 8671. $$

Also
$$ {}_{xy}R^2_z = \frac{r^2_{yz} + r^2_{xz} - 2r_{yz}r_{xz}r_{xy}}{1 - r^2_{xy}} $$
$$ = \cdot 7233, $$

and
$$ {}_{xy}R_z = \cdot 8505. $$

We see that the regression of age on height and weight is very far from planar. The difference

$$ {}_{xy}H^2_z - {}_{xy}R^2_z = \cdot 0286. $$

We have at present* no formula for the probable error of the multiple correlation ratio, but some idea of its magnitude may be obtained from Blakeman's formula† for the probable error of ${}_x\eta^2_y - r^2_{xy}$:

$$ \frac{{}_x\eta^2_y - r^2_{xy}}{\text{Probable error of } ({}_x\eta^2_y - r^2_{xy})} = \frac{\sqrt{N} \cdot \frac{1}{2} \cdot \sqrt{\eta^2 - r^2}}{\cdot 67449\sqrt{1 + (1-\eta^2)^2 - (1-r^2)^2}}. $$

* Since writing the above, the author has obtained a formula for the p.e. of ${}_{xy}H_z$. It is very similar to Pearson's well known formula for the p.e. of the simple η.

† J. Blakeman. *Biometrika*, Vol. IV, p. 332.

With the above values of ${}_{xy}H^2{}_z$ and ${}_{xy}R^2{}_z$, the right-hand side of Blakeman's formula reduces to 13·47, so that on this assumption ${}_{xy}H^2{}_z - {}_{xy}R^2{}_z$ is 13 or 14 times greater than its probable error.

If we examine the regression of weight on height and age, we find in the same way that:

$$ {}_{zx}H^2{}_y = \cdot 8634, \qquad {}_{zx}H_y = \cdot 9292, $$
$$ {}_{zx}R^2{}_y = \cdot 8427, \qquad {}_{zx}R_y = \cdot 9180, $$
$$ {}_{zx}H^2{}_y - {}_{zx}R^2{}_y = \cdot 0207. $$

In this case too the regression is significantly non-planar, Blakeman's formula reducing to 11·43 so that ${}_{zx}H^2{}_y - {}_{zx}R^2{}_y$ is more than 11 times greater than its probable error.

Physically ${}_{zx}H_y$ is more interesting than ${}_{xy}H_z$. A knowledge of the former enables us to construct by means of equation (53) of my Part I, a formula predicting the mean weight of a group of boys of given age and height. Such a formula would be an alternative solution to the one given by Table α' on p. 300 of the paper by Miss Elderton referred to above.

6. We may consider now equation (70) of Part I which gives the approximate age of ${}_{xy}H^2{}_z - {}_{xy}R^2{}_z$ in terms of constants *all derivable* from Tables XI, XII, XIII, i.e. constants whose determination does not require a detailed knowledge of the distribution. The equation is:

$$ ({}_{xy}H^2{}_z - {}_{xy}R^2{}_z)\,\frac{q_{x^2y^2}-1}{q_{x^2y^2}-r^2_{xy}} $$
$$ = \frac{r_{xy}q_{x^2y}-q_{xy^2}}{q_{x^2y}r_{xz}-q_{xy^2}r_{yz}} \cdot \frac{r_{yz}-r_{xz}r_{xy}}{1-r^2_{xy}} \left[({}_y\eta^2{}_z - r^2_{zy}) - \left(\frac{r_{xz}-r_{yz}r_{xy}}{1-r^2_{xy}}\right)^2 ({}_y\eta^2{}_x - r^2_{xy}) \right] $$
$$ + \frac{r_{xy}q_{xy^2}-q_{x^2y}}{q_{xy^2}r_{yz}-q_{x^2y}r_{xz}} \cdot \frac{r_{xz}-r_{yz}r_{xy}}{1-r^2_{xy}} \left[({}_x\eta^2{}_z - r^2_{zx}) - \left(\frac{r_{yz}-r_{xz}r_{xy}}{1-r^2_{xy}}\right)^2 ({}_x\eta^2{}_y - r^2_{xy}) \right] $$
$$ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \ldots\ldots\ldots(70). $$

When ${}_x\beta_1$, ${}_y\beta_1$* are very small and ${}_y\eta_x$, ${}_x\eta_y$ and r_{xy} are nearly equal, we use instead of (70) the following equations:

$$ {}_{xy}H^2{}_z - {}_{xy}R^2{}_z = \frac{q_{x^2y^2}-r^2_{xy}}{q_{x^2y^2}-1}\,({}_y\eta^2{}_z - r^2_{zy}) \quad \ldots\ldots\ldots\ldots\ldots\ldots(71), $$

$$ {}_{xy}H^2{}_z - {}_{xy}R^2{}_z = \frac{q_{x^2y^2}-r^2_{xy}}{q_{x^2y^2}-1}\,({}_x\eta^2{}_z - r^2_{zx}) \quad \ldots\ldots\ldots\ldots\ldots\ldots(72), $$

or approximately

$$ {}_{xy}H^2{}_z - {}_{xy}R^2{}_z = \frac{1+r^2_{xy}}{2r^2_{xy}}\,({}_y\eta^2{}_z - r^2_{zy}) \quad \ldots\ldots\ldots\ldots\ldots\ldots(71)', $$

$$ {}_{xy}H^2{}_z - {}_{xy}R^2{}_z = \frac{1+r^2_{xy}}{2r^2_{xy}}\,({}_x\eta^2{}_z - r^2_{zx}) \quad \ldots\ldots\ldots\ldots\ldots\ldots(72)'. $$

* ${}_x\beta_1$ is used to denote the β_1 of the frequencies of the x character.

For the regression of age on height and weight, the values of ${}_{xy}H^2_z - {}_{xy}R^2_z$ and of ${}_{xy}H^2_z$, given by the preceding equations, are tabulated below:

TABLE XVI.

Source	${}_{xy}H^2_z - {}_{xy}R^2_z$	${}_{xy}H^2_z$
(70)	·0303	·7537
(71)	·0392	·7625
(71)′	·0381	·7615
(72)	·0273	·7507
(72)′	·0266	·7499
Direct Calculation	·0286	·7519

We notice that (72) and (72)′ give much better values than (71) and (71)′. This is to be expected for ${}_x\beta_1$ is very small with a value of ·0003, while ${}_y\beta_1$ is quite appreciable with a value of ·3370, and since r_{zy}, ${}_y\eta_x$, ${}_x\eta_y$ are pretty nearly equal (cf. Table XIV), equations (72) and (72)′ may be used to replace (70).

Another point of interest is that there is very little to choose between the values given by the cumbersome formula (70) and those given by (72) and (72)′.

If we now consider the regression of weight on height and age, we find that equation (70) is unsuitable for the calculation of ${}_{zx}H^2_y - {}_{zx}R^2_y$.

For in this case

$$
\begin{aligned}
{}_x\beta_1 &= \cdot0003,\\
{}_z\beta_1 &= \cdot0057,\\
{}_x\eta_z &= \cdot8448,\\
{}_z\eta_x &= \cdot8314,\\
r_{zx} &= \cdot8305,
\end{aligned}
$$

and we expect the right-hand side of (70) to be indeterminate (cf. Part I, p. 406). We find

$$
\begin{aligned}
r_{zx}q_{zx^2} - q_{z^2x} &= -\cdot007104,\\
r_{zx}q_{z^2x} - q_{zx^2} &= -\cdot002767,\\
\text{and} \quad r_{xy}q_{z^2x} - r_{zy}q_{zx^2} &= -\cdot000175.
\end{aligned}
$$

Thus the right-hand side of equation (70) depends on the ratio of quantities which practically vanish when their probable errors are taken into account.

If, nevertheless, (70) is employed, it gives ${}_{zx}H^2_y - {}_{zx}R^2_y = \cdot1580$, and since ${}_{zx}R^2_y = \cdot8427$, we get ${}_{zx}H^2_y = 1\cdot0007$!, i.e. a value just greater than unity*.

The values of ${}_{zx}H^2_y$ obtained from (71) and (72) are tabulated below:

TABLE XVII.

Source	${}_{zx}H^2_y - {}_{zx}R^2_y$	${}_{zx}H^2_y$
(71)	·0054	·8481
(72)	·0203	·8630
(71)′	·0048	·8474
(72)′	·0177	·8604
Direct Calculation	·0207	·8634

* Of course by an amount < p.e. of ${}_{zx}R_y$.

In this example also as ${}_x\beta_1$ is much nearer to zero than ${}_z\beta_1$ we get better values from (72) and (72)′ than from (71) and (71)′.

We may conclude from these numerical cases that, if

(i) ${}_x\beta_1$ and ${}_y\beta_1$ are fairly small, and

(ii) the regressions of y on x and that of x on y do not depart widely from linearity, we get very fair values for ${}_{xy}H^2{}_z$ from an equation of type

$$ {}_{xy}H^2{}_z - {}_{xy}R^2{}_z = \frac{1+r^2{}_{xy}}{2r^2{}_{xy}}\,({}_y\eta^2{}_z - r^2{}_{zy}), $$

and a very good value from one of type

$$ {}_{xy}H^2{}_z - {}_{xy}R^2{}_z = \frac{q_{x^2y^2} - r^2{}_{xy}}{q_{x^2y^2} - 1}\,({}_y\eta^2{}_z - r^2{}_{zy}), $$

and that it is better to use the equation which corresponds to the smaller β_1 than to use the mean of the values given by the two equations of each type, i.e. if ${}_y\beta_1$ is smaller than ${}_x\beta_1$ we use the formula in terms of ${}_y\eta^2{}_z - r^2{}_{zy}$ in preference to the one involving ${}_x\eta^2{}_z - r^2{}_{zx}$ and in preference to the mean of the two.

If the conditions (i) and (ii) are not satisfied, we use equation (70), which requires more labour than (71) or (72), but is much easier than a direct calculation.

7*. It is a matter of interest to see whether the regression surface of age on height is of the type considered in Part I of this paper. The regression surface being

$$ \frac{\bar{z}_{xy} - \bar{z}}{\sigma_z} = d + \frac{a\,(x-\bar{x})}{\sigma_x} + \frac{b\,(y-\bar{y})}{\sigma_y} + \frac{c\,(x-\bar{x})\,(y-\bar{y})}{\sigma_x\sigma_y} + \frac{e\,(x-\bar{x})^2}{\sigma_x{}^2} + \frac{f\,(y-\bar{y})^2}{\sigma_y{}^2} \quad \text{.....(28).} $$

The solution of equations (29)–(34) is:

$$ \begin{aligned} d &= \cdot 060374, & c &= -\cdot 0940467, \\ a &= \cdot 360742, & e &= \cdot 072020, \\ b &= \cdot 547960, & f &= -\cdot 040709. \end{aligned} $$

Here c is decidedly smaller than a and b, but e and f are of the same order as c. This seems to suggest that our formulae have a wider range than the type of surface by which they were suggested. If we solve equations (36) to (39) (Part I) in which the coefficients e and f are assumed to be zero, we obtain

$$ \begin{aligned} d &= \cdot 060172, & b &= \cdot 512257, \\ a &= \cdot 376024, & c &= -\cdot 066306. \end{aligned} $$

On substituting these values in (59) and (60), we find

$$ {}_{xy}H^2{}_z = \cdot 7470, $$

or

$$ {}_{xy}H^2{}_z = \cdot 7534. $$

Now equations (59) and (60) were based on the assumption that the arrays of y's for constant x and the arrays of x's for constant y are homoscedastic.

* This section differs from the preceding—it does not deal with genuine approximation formulae—for in the determination of the constants b, c, d we use q_{xyz}, i.e. the *detailed table*. It illustrates some other points in Part I.

It is easy to remove this restriction. As on p. 402, we write

$$ {}_{xy}H^2_z - {}_y\eta^2_z = S_y\left\{\left(a + \frac{cy}{\sigma_y}\right)^2 \frac{\sigma^2 x_y}{\sigma^2_x}\frac{n_y}{N}\right\}$$
$$= a^2(1 - {}_y\eta^2_x) + 2ac\,S_y\left(\frac{y}{\sigma_y}\frac{\sigma^2 x_y}{\sigma^2_x}\frac{n_y}{N}\right) + c^2 S_y\left(\frac{y^2}{\sigma^2_y}\frac{\sigma^2 x_y}{\sigma^2_x}\frac{n_y}{N}\right) \quad \ldots\ldots(59)'.$$

If the x arrays are homoscedastic $\dfrac{\sigma^2 x_y}{\sigma^2_x} = 1 - {}_x\eta^2_y$ and we get (59).

Similarly equation (60), corrected for heteroscedasticity, becomes

$$ {}_{xy}H^2_z - {}_x\eta^2_z = b^2(1 - {}_x\eta^2_y) + 2bcS_x\left(\frac{x}{\sigma_x}\frac{\sigma^2 y_z}{\sigma^2_y}\frac{n_x}{N}\right) + c^2 S_x\left(\frac{x^2}{\sigma^2_x}\frac{\sigma^2 y_z}{\sigma^2_y}\frac{n_x}{N}\right) \quad \ldots\ldots\ldots(60)'.$$

Applying these corrections to the case of ${}_{xy}H^2_z$, we find

$$ {}_{xy}H^2_z - \cdot 724535 = \cdot 0221969 + \cdot 0003345 + \cdot 0006875 \quad \ldots\ldots\ldots\ldots(59)',$$

or
$$ {}_{xy}H^2_z = \cdot 7478$$

and
$$ {}_{xy}H^2_z - \cdot 713674 = \cdot 0424845 - \cdot 0053862 + \cdot 0010073 \quad \ldots\ldots\ldots(60)',$$

or
$$ {}_{xy}H^2_z = \cdot 7518,$$

showing the separate contributions of the various terms.

The agreement of (60)′ with the empirical value ·7519 is remarkably close.

For the regression of weight on height and age, equations (36) to (39) give the values

$$a = \cdot 24570, \qquad b = \cdot 70004, \qquad c = \cdot 12160,$$

if the regression hyperboloid be written in the form:

$$\frac{\bar{y}_{xz} - \bar{y}}{\sigma_y} = d + a\frac{z - \bar{z}}{\sigma_z} + b\frac{x - \bar{x}}{\sigma_x} + c\frac{(x - \bar{x})(z - \bar{z})}{\sigma_x \sigma_z}.$$

The values given by the equations corresponding to (59) and (60) are

$$ {}_{xz}H^2_y = {}_x\eta^2_y + (a^2 + c^2)(1 - {}_x\eta^2_z) = \cdot 838012 + \cdot 021519 = \cdot 8595,$$
$$ {}_{xz}H^2_y = {}_z\eta^2_y + (b^2 + c^2)(1 - {}_z\eta^2_x) = \cdot 693989 + \cdot 15580 = \cdot 8498,$$

while the equations corresponding to (59)′ and (60)′ give

$$ {}_{xz}H^2_y = \cdot 838012 + \cdot 0172780 + \cdot 00011322 + \cdot 0034254 = \cdot 8588,$$

and
$$ {}_{xz}H^2_y = \cdot 6939891 + \cdot 15128 + \cdot 0048971 + \cdot 0091002 = \cdot 8593,$$

the empirical value of ${}_{xz}H^2_y$ being ·8634.

These approximations are not quite so good; they are bound up rather closely with the assumed hyperboloidal form of the regression surface, while Mr Soper has shown that in the case of weight on age and height, a good fit is obtained by a surface of the form:

$$\bar{y}_{xz} = a_0 + a_1 z + a_2 z^2 + (b_0 + b_1 z + b_2 z^2)\,x^*.$$

8. Mr M. Greenwood, Jr, has kindly drawn my attention to a joint paper, entitled "A Study of Index Correlations," published by Mr J. W. Brown, Miss Frances Wood and himself in the *Journal of the Royal Statistical Society*†.

* Elderton's paper, *loc. cit.* p. 294.

† Vol. LXXVII, Part III, February, 1914, pp. 317–346.

The authors give the value of a generalized H, obtained by direct calculation in two or three cases. The material they deal with is the distribution of births, deaths and populations in 1000 English registration sub-districts, and also of the birth- and death-rates and populations in these districts. The material is highly heterogeneous and exceedingly high values of β_1 occur. Thus, β_1 for deaths is 16·0205 and for death rates is 61·5392.

Unfortunately in their table on page 341 of the values of the simple η's, the authors fail to discriminate between ${}_y\eta_x$ and ${}_x\eta_y$.

Mr Greenwood, at my request, was good enough to consult his MS. notes and sent me an identification table of the η's, but a recalculation of the η of populations on births and the η of populations on deaths has shown that errors have crept into this identification table.

When amended this table is as follows:

$$
\begin{aligned}
{}_x\eta_y &= \cdot 9501,\\
{}_y\eta_x &= \cdot 9358,\\
{}_x\eta_z &= \cdot 9241,\\
{}_z\eta_x &= \cdot 7855,\\
{}_z\eta_y &= \cdot 7527,\\
{}_y\eta_z &= \cdot 8871,
\end{aligned}
$$

where x = population, y = births, z = deaths.

With these values our equation (70) gives

$$
{}_{xy}H^2{}_z - {}_{xy}R^2{}_z = \cdot 2691,
$$

or
$$
{}_{xy}H^2{}_z = \cdot 8814,
$$

and
$$
{}_{xy}H_z = \cdot 9388.
$$

The authors give as the empirical value ${}_{xy}H_z = \cdot 9384$.

As regards the use of the simpler formulae, the only β_1 of reasonably small value is β_1 of population which equals ·0886.

We put
$$
{}_{xy}H^2{}_z - {}_{xy}R^2{}_z = \frac{1 + r^2{}_{xy}}{2r^2{}_{xy}} ({}_x\eta^2{}_z - r^2{}_{xz}),
$$

or
$$
{}_{xy}H^2{}_z - (\cdot 7825)^2 = \frac{1 + (\cdot 9345)^2}{2\,(\cdot 9345)^2} [(\cdot 9241)^2 - (\cdot 7822)^2],
$$

or
$$
\begin{aligned}
{}_{xy}H^2{}_z &= \cdot 87200,\\
{}_{xy}H_z &= \cdot 9338.
\end{aligned}
$$

Considering the wide divergence in the results obtained by the authors when dealing with the 1000 sub-districts and with 999 of them* it is evident that the use of the above simple formula would have been sufficient for their purpose.

* *l.c.*, p. 343: ${}_{\text{population and births}}H_{\text{deaths}} = \cdot 9384$ for 1000 sub-districts, and $= \cdot 9213$ for 999 sub-districts.

GLASGOW SCHOOL BOYS—GROUP A.

TABLE I. *Age* 4·5 *to* 5·5 *years.*

Weight in lbs.

Height in inches	*24—28*	*29—33*	*34—38*	*39—43*	*44—48*	*49—53*	Totals
30—32	2	—	—	—	—	—	2
33—35	5	15	5	—	—	—	25
36—38	1	18	72	8	—	—	99
39—41	—	5	87	90	7	1	190
42—44	—	—	4	35	21	5	65
45—47	—	—	1	—	2	—	3
Totals	8	38	169	133	30	6	384

TABLE II. *Age* 5·5 *to* 6·5 *years.*

Weight in lbs.

Height in inches	*24—28*	*29—33*	*34—38*	*39—43*	*44—48*	*49—53*	*54—58*	*59—63*	*64—68*	Totals
30—32	1	1	—	—	—	—	—	—	—	2
33—35	4	17	2	—	—	—	—	—	—	23
36—38	1	36	92	14	1	1	1	—	—	146
39—41	—	16	191	258	38	1	—	—	1	505
42—44	—	2	27	232	178	28	2	—	—	469
45—47	—	—	1	12	39	37	6	—	—	95
48—51	—	—	—	1	1	—	2	—	—	4
Totals	6	72	313	517	257	67	11	—	1	1244

TABLE III. *Age* 6·5 *to* 7·5 *years.*

Weight in lbs.

Height in inches	*24—28*	*29—33*	*34—38*	*39—43*	*44—48*	*49—53*	*54—58*	*59—63*	Totals
30—32	1	1	—	1	—	—	—	—	3
33—35	—	8	6	2	—	—	—	—	16
36—38	—	6	37	10	2	—	—	—	55
39—41	—	4	93	178	35	2	—	—	312
42—44	—	—	22	220	294	61	3	1	601
45—47	1	1	—	21	133	138	38	—	332
48—50	—	—	—	2	—	13	12	4	31
51—53	—	—	—	—	—	1	—	1	2
Totals	2	20	158	434	464	215	53	6	1352

TABLE IV. *Age* 7·5 *to* 8·5 *years.*

Weight in lbs.

Height in inches	*29—33*	*34—38*	*39—43*	*44—48*	*49—53*	*54—58*	*59—63*	*64—68*	*69—73*	*74—78*	Totals
33—35	2	3	1	—	—	—	—	—	—	—	6
36—38	—	13	15	3	1	—	—	—	—	—	32
39—41	—	38	63	18	2	—	—	—	—	—	121
42—44	1	13	145	257	74	1	—	—	—	—	491
45—47	—	2	36	204	283	89	4	1	—	—	619
48—50	—	—	1	13	82	86	40	7	—	—	229
51—53	—	—	—	6	9	4	9	1	1	1	31
54—56	—	—	—	—	—	2	—	—	—	—	2
57—59	—	—	—	—	2	1	1	—	—	—	4
Totals	3	69	261	501	453	183	54	9	1	1	1535

TABLE V. *Age* 8·5 *to* 9·5 *years.*

Weight in lbs.

Height in inches	*29—33*	*34—38*	*39—43*	*44—48*	*49—53*	*54—58*	*59—63*	*64—68*	*69—73*	*74—78*	*79—83*	Totals
30—32	1	—	—	—	—	—	—	—	—	—	—	1
33—35	—	—	—	—	—	—	—	—	—	—	—	—
36—38	2	2	1	1	1	1	—	—	—	—	—	8
39—41	—	4	21	15	2	1	—	—	—	—	—	43
42—44	—	3	48	90	41	5	—	—	—	—	—	187
45—47	—	2	18	150	286	118	9	—	—	—	—	583
48—50	—	—	3	10	118	217	98	11	1	—	—	458
51—53	—	—	—	1	5	20	41	23	9	—	—	99
54—56	—	—	—	1	5	2	1	3	4	1	1	18
57—59	—	—	—	—	1	—	—	—	—	1	—	2
60—62	—	—	—	—	—	1	1	—	1	—	—	3
Totals	3	11	91	268	459	365	150	37	15	2	1	1402

TABLE VI. *Age* 9·5 *to* 10·5 *years.*

Weight in lbs.

Height in inches	*34—38*	*39—43*	*44—48*	*49—53*	*54—58*	*59—63*	*64—68*	*69—73*	*74—78*	*79—83*	Totals
33—35	—	—	1	—	—	—	—	—	—	—	1
36—38	2	3	—	1	1	1	—	—	—	—	8
39—41	1	7	6	2	—	—	—	—	—	—	16
42—44	2	14	27	23	4	—	—	—	—	—	70
45—47	—	5	54	148	93	10	1	—	—	—	311
48—50	—	3	3	127	281	159	35	5	—	—	613
51—53	—	1	—	7	42	130	108	27	7	1	323
54—56	—	—	—	—	—	3	11	10	5	1	30
57—59	—	—	—	—	1	2	—	—	1	—	4
60—62	—	—	—	—	2	—	—	—	—	—	2
63—65	—	—	—	—	—	—	1	—	—	—	1
Totals	5	33	91	308	424	305	156	42	13	2	1379

TABLE VII. *Age* 10·5 *to* 11·5 *years.*

Weight in lbs.

Height in inches	*34—38*	*39—43*	*44—48*	*49—53*	*54—58*	*59—63*	*64—68*	*69—73*	*74—78*	*79—83*	*84—88*	Totals
33—35	—	—	—	—	1	—	—	—	—	—	—	1
36—38	1	—	—	—	—	—	—	—	—	—	—	1
39—41	—	—	3	1	—	—	—	1	—	—	—	5
42—44	1	2	8	12	1	—	—	—	—	—	—	24
45—47	—	3	16	65	55	10	2	—	—	—	—	151
48—50	—	1	5	40	184	161	45	11	—	—	—	447
51—53	—	—	—	7	49	166	148	81	12	2	—	465
54—56	—	—	—	—	1	13	42	56	45	8	3	168
57—59	—	—	—	1	—	2	—	2	1	3	5	14
60—62	—	—	—	—	—	—	2	—	—	—	—	2
Totals	2	6	32	126	291	352	239	151	58	13	8	1278

TABLE VIII. *Age* 11·5 *to* 12·5 *years.*

Weight in lbs.

Height in inches	*34—38*	*39—43*	*44—48*	*49—53*	*54—58*	*59—63*	*64—68*	*69—73*	*74—78*	*79—83*	*84—88*	*89—93*	*94—98*	*99—103*	Totals
36—38	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
39—41	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
42—44	—	1	—	3	1	—	—	—	—	—	—	—	—	—	5
45—47	—	—	3	23	22	2	4	—	—	—	—	—	—	—	54
48—50	—	—	3	22	83	101	36	11	—	2	—	—	—	—	258
51—53	—	—	—	—	41	125	168	108	26	4	2	—	—	—	474
54—56	—	—	—	—	5	14	64	100	97	33	6	2	—	—	321
57—59	—	—	—	—	—	1	—	3	16	20	10	5	1	1	57
60—62	—	—	—	—	—	—	—	1	—	1	1	—	—	1	4
63—65	—	—	—	—	—	—	1	—	—	1	1	—	—	—	3
Totals	1	1	6	49	152	243	273	223	139	61	20	7	1	2	1178

TABLE IX. *Age* 12·5 *to* 13·5 *years.*

Weight in lbs.

Height in inches	*44—48*	*49—53*	*54—58*	*59—63*	*64—68*	*69—73*	*74—78*	*79—83*	*84—88*	*89—93*	*94—98*	*99—103*	*104—108*	*109—113*	Totals
42—44	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
45—47	2	6	12	3	—	—	—	—	1	—	—	—	—	—	24
48—50	2	8	35	60	20	6	3	—	—	—	—	—	—	—	134
51—53	1	2	12	80	142	112	51	14	6	1	—	—	—	—	421
54—56	—	—	1	13	59	129	138	50	17	6	1	—	1	—	415
57—59	—	—	1	—	3	12	38	42	44	15	15	1	—	—	171
60—62	—	—	—	—	—	—	4	2	2	14	5	5	—	1	33
63—65	—	—	—	—	—	—	—	—	—	1	—	2	—	—	3
Totals	5	17	61	156	224	259	234	108	70	37	21	8	1	1	1202

TABLE X. *Age 13·5 to 14·5 years.*

Weight in lbs.

Height in inches	*44—48*	*49—53*	*54—58*	*59—63*	*64—68*	*69—73*	*74—78*	*79—83*	*84—88*	*89—93*	*94—98*	*99—103*	*104—108*	*109—113*	*114—118*	*119—123*	Totals
42—44	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
45—47	—	2	3	1	—	—	—	—	—	—	—	—	—	—	—	—	6
48—50	1	1	5	7	7	1	—	—	—	—	—	—	—	—	—	—	22
51—53	—	2	3	16	31	36	9	1	—	—	—	—	—	—	—	—	98
54—56	—	—	—	7	27	41	49	27	6	2	2	—	—	—	—	—	161
57—59	—	—	1	—	—	7	19	28	28	16	7	3	—	—	—	—	109
60—62	—	—	—	—	—	—	1	1	5	4	4	5	5	—	—	—	25
63—65	—	—	—	—	—	—	—	—	1	—	2	1	1	—	—	1	6
Totals	1	6	12	31	65	85	78	57	40	22	15	9	6	—	—	1	428

TABLE XI. *Correlation Table of Heights and Weights.*

Height in inches.

Weight in lbs.	*30—32*	*33—35*	*36—38*	*39—41*	*42—44*	*45—47*	*48—50*	*51—53*	*54—56*	*57—59*	*60—62*	*63—65*	Totals
24—28	4	9	2	—	—	1	—	—	—	—	—	—	16
29—33	3	42	62	25	3	1	—	—	—	—	—	—	136
34—38	—	16	220	414	72	6	—	—	—	—	—	—	728
39—43	1	3	51	617	697	95	11	1	—	—	—	—	1476
44—48	—	1	7	122	875	603	38	8	1	—	—	—	1655
49—53	—	—	4	12	249	988	411	33	5	4	—	—	1706
54—58	—	1	3	1	17	436	905	171	11	4	3	—	1552
59—63	—	—	1	—	1	39	630	568	51	6	1	—	1297
64—68	—	—	—	1	—	8	161	621	206	3	2	2	1004
69—73	—	—	—	1	—	—	35	374	340	24	2	—	776
74—78	—	—	—	—	—	—	3	106	335	76	5	—	525
79—83	—	—	—	—	—	—	2	22	120	93	4	1	242
84—88	—	—	—	—	—	1	—	8	32	87	8	2	138
89—93	—	—	—	—	—	—	—	1	10	36	18	1	66
94—98	—	—	—	—	—	—	—	—	3	23	9	2	37
99—103	—	—	—	—	—	—	—	—	—	5	11	3	19
104—108	—	—	—	—	—	—	—	—	1	—	5	1	7
109—113	—	—	—	—	—	—	—	—	—	—	1	—	1
114—118	—	—	—	—	—	—	—	—	—	—	—	—	—
119—123	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	8	72	350	1193	1914	2178	2196	1913	1115	361	69	13	11382

TABLE XII. *Correlation Table of Height and Age.*

Age in years

Height in inches	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	Totals
30—32	2	2	3	—	1	—	—	—	—	—	8
33—35	25	23	16	6	—	1	1	—	—	—	72
36—38	99	146	55	32	8	8	1	1	—	—	350
39—41	190	505	312	121	43	16	5	1	—	—	1193
42—44	65	469	601	491	187	70	24	5	1	1	1914
45—47	3	95	332	619	583	311	151	54	24	6	2178
48—50	—	4	31	229	458	613	447	258	134	22	2196
51—53	—	—	2	31	99	323	465	474	421	98	1913
54—56	—	—	—	2	18	30	168	321	415	161	1115
57—59	—	—	—	4	2	4	14	57	171	109	361
60—62	—	—	—	—	3	2	2	4	33	25	69
63—65	—	—	—	—	—	1	—	3	3	6	13
Totals	384	1244	1352	1535	1402	1379	1278	1178	1202	428	11382

TABLE XIII. *Correlation Table of Age and Weight.*

Weight in lbs. \ Age in years	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	Totals
24—28	8	6	2	—	—	—	—	—	—	—	16
29—33	38	72	20	3	3	—	—	—	—	—	136
34—38	169	313	158	69	11	5	2	1	—	—	728
39—43	133	517	434	261	91	33	6	1	—	—	1476
44—48	30	257	464	501	268	91	32	6	5	1	1655
49—53	6	17	215	453	459	308	126	49	17	6	1706
54—58	—	11	53	183	365	424	291	152	61	12	1552
59—63	—	—	6	54	150	305	352	243	156	31	1297
64—68	—	1	—	9	37	156	239	273	224	65	1004
69—73	—	—	—	1	15	42	151	223	259	85	776
74—78	—	—	—	1	2	13	58	139	234	78	525
79—83	—	—	—	—	1	2	13	61	108	57	242
84—88	—	—	—	—	—	—	8	20	70	40	138
89—93	—	—	—	—	—	—	—	7	37	22	66
94—98	—	—	—	—	—	—	—	1	21	15	37
99—103	—	—	—	—	—	—	—	2	8	9	19
104—108	—	—	—	—	—	—	—	—	1	6	7
109—113	—	—	—	—	—	—	—	—	1	—	1
114—118	—	—	—	—	—	—	—	—	—	—	—
119—123	—	—	—	—	—	—	—	—	—	1	1
Totals	384	1244	1352	1535	1402	1379	1278	1178	1202	428	11382

TABLE XV. *Mean Ages of Height and Weight Arrays.*

x	y	z_{xy}	x	y	z_{xy}	x	y	z_{xy}	x	y	z_{xy}	x	y	z_{xy}
−6	−6	−4·25	−3	0	−1	0	−1	−0·508516	2	2	2·194174	4	3	0·5
	−5	−2·666667		2	−4		0	0·046409		3	2·361764		4	3·2
	−3	−3		3	1		1	0·604762		4	2·531343		5	3
−5	−6	−4·555556	−2	−5	−3·333333		2	1·118012		5	2·758333		6	3·5
	−5	−4·071428		−4	−3·083333		3	1·542857		6	2·812500		7	3·222222
	−4	−3·5625		−3	−3·008608		4	3		7	3		8	3·444444
	−3	−2·666667		−2	−2·6222857		5	2		8	3·666667		9	3·363636
	−2	0		−1	−1·943775	1	−3	0		10	3		10	4
	0	1		0	−1·235294		−2	−1·25	3	−1	−1		11	3
−4	−6	−4·5		1	−3		−1	−0·151515		0	1·25	5	2	1
	−5	−4·096774	−1	−6	−3		0	0·883041		1	0·333333		5	2
	−4	−3·927273		−5	−3		1	1·158450		2	3		6	3
	−3	−3·078431		−4	−2·5		2	1·624798		3	3		7	3
	−2	−2·428571		−3	−2·084210		3	2·048128		4	2·921052		8	4
	−1	−1·75		−2	−1·815920		4	2·367924		5	3·021505		9	3·333333
	0	−1·666667		−1	−1·292510		5	2·545455		6	3·091954		10	4
	1	0		0	−0·658257		6	2·75		7	3·305555		13	4
−3	−5	−4·04		1	0·256410		7	3		8	3·260869			
	−4	−3·763285		2	1	2	−2	−1		9	3·4			
	−3	−3·505673		6	3		−1	−1	4	0	−0·333333			
	−2	−2·786885	0	−3	−1·272727		0	0·727273		1	−1			
	−1	−1·5		−2	−0·5		1	2·098039		2	1			

CORRELATION OF ANTHROPOMETRICAL MEASUREMENTS IN CAIRO-BORN NATIVES*

By MYER M. ORENSTEEN

(*Ministry of Public Works, Egypt*)

1. *The Material.* Since 1910, owing to the completion of an anthropometrical study undertaken by the Egyptian Government under the direction of Mr J. I. Craig, now Controller-General of the Statistical Department, a series of measurements of about ten thousand Egyptian natives has been filed in the Survey Department. It has been thought of interest to take this opportunity to re-examine the material in order to determine the correlation between the different organs of the native body available.

Referring to the nature of the characters considered in the paper, it will be necessary to remember here that the data belong to anthropometrical slips of prisoners, taken from the Cairo Anthropometrical Bureau. The slips have been drawn absolutely at random without any particular character of selection.

To Mr J. I. Craig, the writer addresses his best thanks for kindly allowing him full access to the material, and for valuable advice.

2. *The Data.* The measurements available and dealt with in the paper are:

(*L.*) *Length of the Head.* This is the shortest distance between the glabella and the occipital point as measured with callipers.

(*B.*) *Breadth of the Head.* This is the maximum breadth of the head. It is measured with callipers just above the back of the ears.

(*L.M.F.*) *Length of the left middle Finger.*

(*L.F.*) *Length of the left Foot.*

(*L.C.*) *Length of the left Cubit.*

(*S.*) *Stature.* This is the total height standing without boots.

From the ten thousand individuals in hand, who are natives of all parts of Egypt up to Aswan, 1191 persons have been selected as being exclusively males born in Cairo. From this selected group a further number of 389 has been

* As the Editor was unable to consult the author of this paper, he has considered it best to alter as little as possible the literary expression.

excluded for being, at the date when measured, youths of under 20 years of age. The remaining 802 individuals providing the final group of measurements dealt with throughout the paper are, therefore, adult male Egyptians born in Cairo. Of this final group 76 per cent. were measured during 1904–1905, the remaining 24 per cent. fall within the years 1901 and 1906.

3. *Social class to which the Natives dealt with belong.* From the fact that these natives have undergone police examination, objection may arise as to whether the measurements of such persons can represent the ordinary native body, or if really they belong to a selected group of individuals, namely the criminal class of natives.

TABLE I.

Movement of the natives measured, their distribution as regards length of imprisonment, and their comparison with migration of ordinary inhabitants.

Length of Imprisonment	Place where migrated to			Total
	Lower Egypt	Cairo Town	Upper Egypt	
One year and less	8·8	71·2	6·8	86·8
Over one year and up to three	0·8	10·5	1·0	12·3
Over three years	0·0	0·8	0·1	0·9
Total	9·6	82·5	7·9	100·0
Ordinary Egyptian inhabitants	6·3	91·1	2·5	100·0

Mr J. I. Craig, writing on the same material, says* "it may be objected that criminality in itself is a determinating factor of selection, but the objection does not hold in Egypt. Here it cannot be said that there exists a definite criminal class, and criminals are rather amateurs than professionals. This state of things is in all probability due to the easy conditions under which the lower class live."

In support of this, I may mention some recent conclusions which Dr Charles Goring comes to†. "Criminals as criminals are not a physically differentiated class of the general community....The physical and mental constitution of both criminal and law-abiding persons of same age, stature, class and intelligence are identical....There is no such thing as an anthropological type."

* "Anthropometry of Modern Egyptians," *Biometrika*, July, 1911.

† *The English Convict: A Statistical Study*, by Charles Goring, M.D.

These statements are suggestive for our material, and some investigation has been thought desirable in order to ascertain how far the former is justified for the Cairo group when considered alone. This investigation was carried out in two directions:

(1) The seriousness of the offence or crime committed. This of course is proportionate to the length of imprisonment.

(2) The social class to which the natives belong. This is traced by looking into the daily occupation of each individual.

TABLE II.

Daily occupation of men measured.

	%		%
Coffee-house keeper ...	9·4	Tailor	2·4
Pedlar	9·2	Cultivator	2·2
Servant	7·9	Baker	2·0
Driver	7·4	Butcher	1·7
Workman	6·9	Cobbler	1·6
Blacksmith	6·4	Barber...	1·3
Vagabond	6·2	Washerman	1·2
Carpenter	5·5	Coal-seller	1·1
Painter	4·8	Water-carrier ...	0·8
Cook	4·0	Shoe-black	0·8
Tobacconist	4·0	Bricklayer	0·6
Clerk	3·1	Student	0·6
Fruiterer	2·7	Printer	0·5
Weaver	2·6	Milkman	0·3
Confectioner	2·5	Watchman	0·3

Table I gives in per cent. the different lengths of imprisonment. This table also exhibits the stability of the group measured when compared with the total mass of the ordinary native male population. The figures of the latter group were taken from the *Census of Egypt in* 1907, where 221,185 individuals have been classified as being male Egyptians born in Cairo. From Table I we observe that most of the men have been condemned to very short punishments. On the whole only nine natives have been registered for murder (manslaughter); 86·8 per cent. have been kept in custody for a period varying between a few days and one year; 12·3 per cent. have been kept in for a length varying from one year and up to three. The remaining 0·9 per cent. have been imprisoned for a period over three years. This state of things is surely due to slight offences done without premeditation.

The figures under the heading "migration" show clearly that most of the Cairenes settle in their native town (as is also the case in the whole country). Out of the Cairo migrating element, ten per cent. prefer to settle down in Lower Egypt while eight per cent. move into Upper Egypt. We further remark that the ordinary

native migrates less than the group measured by the police. The natives of the former selection remaining in the town exceed by 10 per cent. those of the latter. This in all probability is to be explained by the fact that the census figures include children and boys under 20, up to which age there is less tendency to migration than afterwards.

Table II gives a fairly exact idea of the daily occupation or social class to which the men measured belong. From this table we first remark that practically all men have their daily occupation, and that the greater part of them deal directly with the public. The proportion of these is 87 per cent.; the ordinary native element is represented by 52 per cent. Vagabonds and men without distinct occupation come in with 6 per cent.

As a result of the analysis of Tables I and II, we may come to the conclusion that the people dealt with in this paper are not necessarily of the poorest class of the total mass of the Cairo natives. And although they have undergone the police examination they are not necessarily of a selected class of natives, nor are they under special conditions of health.

4. *Age of the People.* As we have already said, all youths under twenty years of age have been excluded. Most of the natives are ignorant of their right age and could easily deviate by ten or twenty years from the truth though with apparent good faith. The age was estimated by the police officer on duty.

The percentage frequency of men per group of age is as follows:

		Census
From 20 to 25 years ...	52 %	25 %
„ 26 to 30 „ ...	28 %	18 %
„ 31 to 40 „ ...	16 %	26 %
„ 41 to 70 „ ...	4 %	32 %

It will be seen that the above grouping eliminates the greater part of accidental errors due to personal equation of the observer. The figures given by the census are said to be very unreliable.

TABLE III.

Means, Standard-deviations and their respective Probable Errors.

Character	$M \pm E_M$	$\sigma^* \pm E_\sigma$
Length of Head, in mm. ...	190·52±0·14	5·90±0·10
Breadth of Head, in mm. ...	144·45±0·11	4·67±0·08
Left Middle Finger, in mm.	114·09±0·15	6·14±0·10
Left Foot, in mm.	257·64±0·30	12·36±0·21
Left Cubit, in mm.	457·51±0·52	21·21±0·36
Stature, in cm.	165·82±0·14	5·93±0·10
Cephalic Index, in % ...	75·82±0·07	2·91±0·05

* Sheppard's corrections have been applied.

5. *Frequency Distributions.* The total number of individuals measured being 802, the measurements were grouped so as to save time and labour.

The grouping of the data has been arranged so as to give approximately eleven sub-ranges of grouping in each case with limits as follows:

Length of the head	...	every 3 millimetres
Breadth of the head	...	„ 3 „
Left middle finger	...	„ 4 „
Left foot	...	„ 7 „
Left cubit ...	...	„ 15 „
Stature	...	„ 4 centimetres

Table III gives the mean, the standard deviation corrected by Sheppard's adjustments, and the corresponding probable error for each character observed. The cephalic index is computed as usual from the ratio of the breadth to the length multiplied by a hundred; this gives a measure as to whether the head is relatively long or broad.

The frequency distributions were further analysed. Sheppard's corrections were applied to all the moments, and the criterion κ for the classification of the curves determined. κ in all cases was less than unity; β_1 and β_2 computed from μ_2, μ_3, μ_4 (the second, third and fourth moments) were practically 0 and 3 respectively for all distributions.

The normal curve of the form $Y = y_0 e^{-x^2/2\sigma^2}$ was therefore chosen.

The theoretical frequencies for every sub-group of each distribution of measurements have been computed from the following formulae:

Length of the head: $f = 162{\cdot}73e^{-x^2/69{\cdot}6132}$,

Breadth of the head: $f = 205{\cdot}50e^{-x^2/43{\cdot}6194}$,

Left middle finger: $f = 208{\cdot}43e^{-x^2/75{\cdot}3952}$,

Left foot: $f = 181{\cdot}20e^{-x^2/305{\cdot}5150}$,

Left cubit: $f = 226{\cdot}30e^{-x^2/899{\cdot}3250}$,

Stature: $f = 215{\cdot}77e^{-x^2/70{\cdot}3072}$,

where f is the frequency for every unit of grouping,

x is the distance from the origin (in these cases the origin and the mean coincide) to the foot of the ordinate to be calculated,

e is the base of the Napierian system of logarithms.

The comparison between the observed frequencies and the theoretical frequencies computed from the above formulae is given in Table IV. χ^2 is calculated from the sum of the squares of the departures of the theoretical values from the observed ones, divided by the corresponding theoretical values. This is shown by the formula

$$\chi^2 = \Sigma\,[(y_O - y_C)^2/y_C].$$

P is the probability that, in another random sample of the same size, a worse fit would be obtained.

TABLE IV.

(*a*) Length of Head

Centre of Group mm.	Observed frequency O	Computed frequency C	$\Delta = O - C$
174	4	3	+ 1
177	8	12	− 4
180	41	33	+ 8
183	69	72	− 3
186	114	122	− 8
189	177	158	+ 19
192	160	158	+ 2
195	98	122	− 24
198	75	73	+ 2
201	39	34	+ 5
204	7	12	− 5
207	10	3	+ 7

$\chi^2 = \Sigma\Delta^2/C = 30{\cdot}3$ $P = {\cdot}0015$

(*b*) Breadth of Head

Centre of Group mm.	Observed frequency O	Computed frequency C	$\Delta = O - C$
131	6	3	+ 3
134	8	17	− 9
137	75	58	+ 17
140	121	130	− 9
143	190	196	− 6
146	202	195	+ 7
149	125	128	− 3
152	55	56	− 1
155	11	16	− 5
158	8	3	+ 3
161	1	0	+ 1

$\chi^2 = \Sigma\Delta^2/C = 19{\cdot}2$ $P = {\cdot}0378$

(*c*) Left Middle Finger

Centre of Group mm.	Observed frequency O	Computed frequency C	$\Delta = O - C$
94·5	1	1	0
98·5	6	8	− 2
102·5	32	35	− 3
106·5	111	97	+ 14
110·5	176	176	0
114·5	215	208	+ 7
118·5	137	161	− 24
122·5	80	82	− 2
126·5	36	27	+ 9
130·5	5	6	− 1
134·5	3	1	+ 2

$\chi^2 = \Sigma\Delta^2/C = 9{\cdot}8$ $P = {\cdot}4590$

(*d*) Left Foot

Centre of Group mm.	Observed frequency O	Computed frequency C	$\Delta = O - C$
225	5	6	− 1
232	19	21	− 2
239	70	58	+ 12
246	111	117	− 6
253	178	169	+ 9
260	167	178	− 11
267	122	136	− 14
274	88	76	+ 12
281	28	30	− 2
288	11	9	+ 2
295	3	2	+ 1

$\chi^2 = \Sigma\Delta^2/C = 8{\cdot}7$ $P = {\cdot}5677$

(*e*) Left Cubit

Centre of Group mm.	Observed frequency O	Computed frequency C	$\Delta = O - C$
395	3	4	− 1
410	20	18	+ 2
425	72	70	+ 2
440	160	161	− 1
455	224	225	− 1
470	186	190	− 4
485	100	97	+ 3
500	26	30	− 4
515	10	6	+ 4
530	0	1	− 1
545	1	0	+ 1

$\chi^2 = \Sigma\Delta^2/C = 3{\cdot}9$ $P = {\cdot}9439$

(*f*) Stature

Centre of Group cm.	Observed frequency O	Computed frequency C	$\Delta = O - C$
149·5	4	5	− 1
153·5	28	25	+ 3
157·5	73	81	− 8
161·5	185	166	+ 19
165·5	212	215	− 3
169·5	167	178	− 11
173·5	87	93	− 6
177·5	31	31	0
181·5	14	7	+ 7
185·5	1	1	0

$\chi^2 = \Sigma\Delta^2/C = 11{\cdot}7$ $P = {\cdot}2341$

FIG. 1. Length of Head. ($y = 202{\cdot}90 \times 2{\cdot}71828^{-x^2/(29{\cdot}6132)}$)

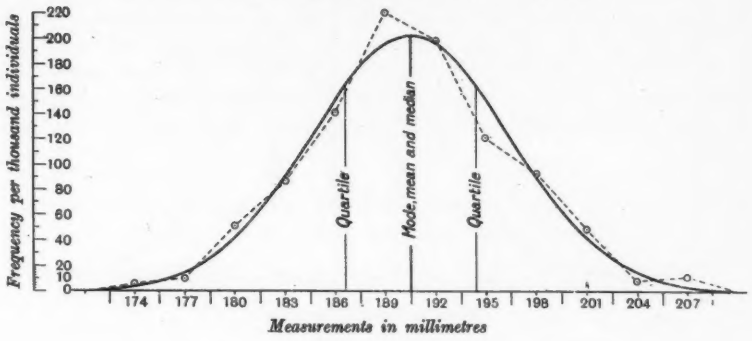


FIG. 2. Breadth of Head. ($y = 256{\cdot}23 \times 2{\cdot}71828^{-x^2/(43{\cdot}6194)}$)

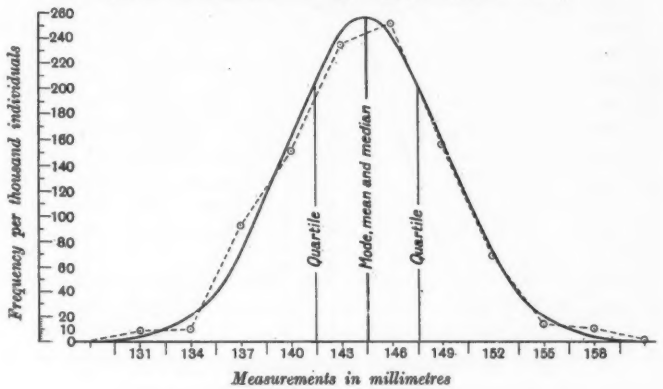


FIG. 3. Left Middle Finger. ($y = 259{\cdot}89 \times 2{\cdot}71828^{-x^2/(75{\cdot}3952)}$)

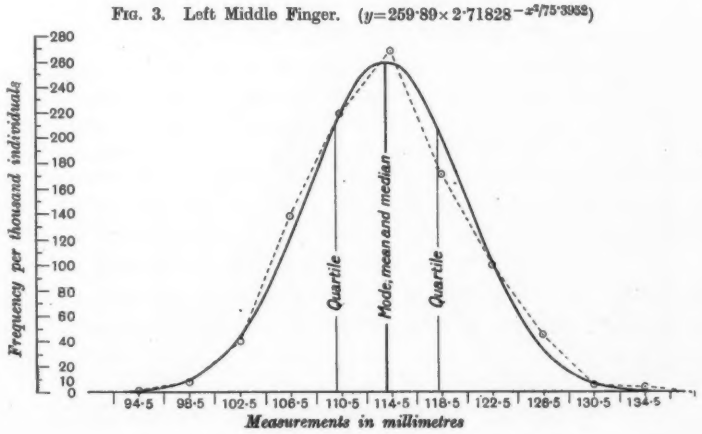


FIG. 4. Left Foot. ($y = 225{\cdot}93 \times 2{\cdot}71828^{-x^2/303{\cdot}515}$)

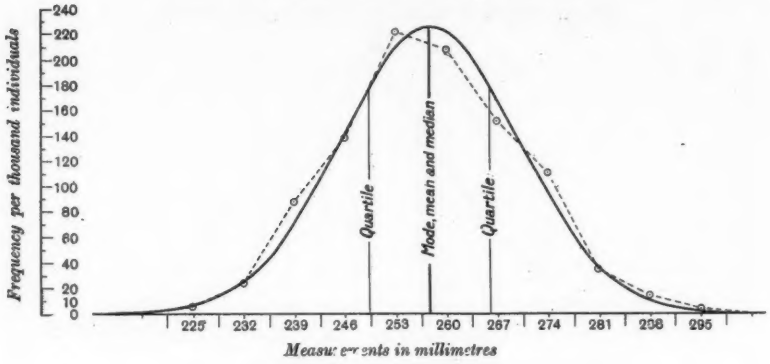


FIG. 5. Left Cubit. ($y = 282{\cdot}17 \times 2{\cdot}71828^{-x^2/890{\cdot}3250}$)

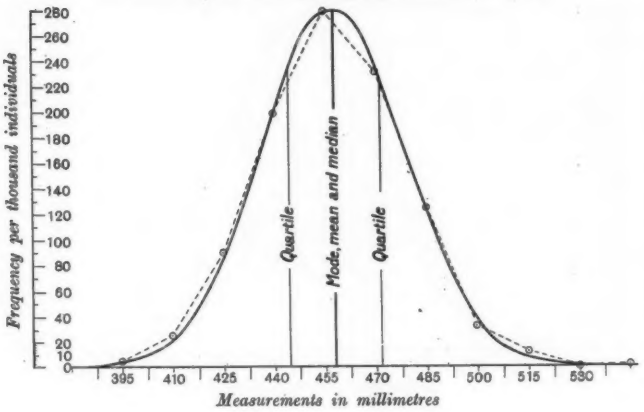
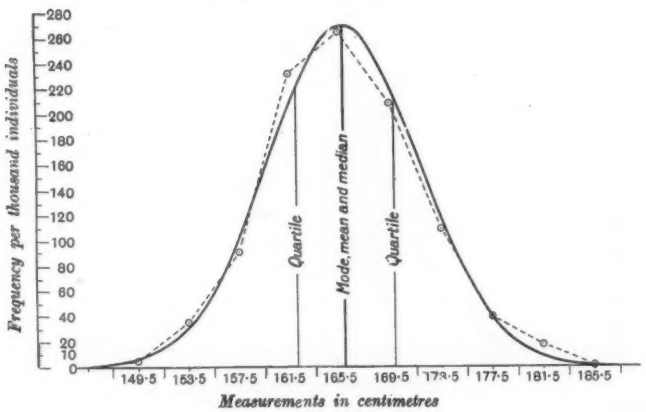


FIG. 6. Stature. ($y = 269{\cdot}04 \times 2{\cdot}71828^{-x^2/70{\cdot}3672}$)



Figures 1 to 6 give the calculated frequency curves. The lengths of the ordinates represent the frequency per thousand of observations in a corresponding group where the ordinate considered is the mean for the unit of grouping. The dotted curves have been drawn from the observed frequencies reduced to the same base. A glance at these figures is sufficient to show that normal curves fit pretty well our statistics. The quartiles have been drawn with the aid of an integraph.

6. *Coefficients of Correlation.* With the same grouping of the data I have constructed correlation tables. These are given in Tables VIII to XIV.

The coefficient of correlation for the length and the breadth of the head was found to be $r = +0{\cdot}244 \pm 0{\cdot}020$. This coefficient is as we should have expected lower than the results from the remaining pairs of characters. Attention may be drawn to the fact that this particular coefficient varies very much from race to race. A comparison with similar results for different races may be of interest here:

♂	
Smith Sound Eskimos ...	$r = +0{\cdot}47 \pm 0{\cdot}08$
Ainos	$r = +0{\cdot}43 \pm 0{\cdot}06$
Naqada Race	$r = +0{\cdot}34 \pm 0{\cdot}05$
Germans	$r = +0{\cdot}29 \pm 0{\cdot}06$
Bavarian Peasants	$r = +0{\cdot}28 \pm 0{\cdot}06$
Cairo Natives	$r = +0{\cdot}24 \pm 0{\cdot}02$
French Peasants	$r = +0{\cdot}13 \pm 0{\cdot}09$
British Columbian Indians ...	$r = +0{\cdot}08 \pm ?$

Taking into consideration the great variation of this coefficient from race to race, the writer is greatly inclined to believe that a correlation between the length and the breadth of the head really exists, but that its variation in value is very probably due to special factors, probably working independently and tending to modify the real measure of relationship. This is being further investigated, and a closer study of such factors is being undertaken.

The results of the computation of Tables VIII to XIV are grouped in Table V. From this table we may deduce the following conclusions: Were all human bodies perfectly similar all the coefficients of correlation between the different organs would be unity; if on the contrary there should exist a complete independency of the organs, the coefficients would be all nil. We may therefore conclude from Table V that we are to expect a tall man to have more often long feet than long fingers and short men will more often have long fingers than long arms. Further, men with long arms will very rarely have short fingers, and such people will more often be men with greater stature than men with long feet, and so on.

Table VI gives the probable error for each coefficient of correlation shown in Table V. They are in all cases very small, which proves our results to be very reliable.

The results shown in Table V have been compared with the like data for European countries, and no serious deviation in the coefficients has been noticed. This tends to confirm the statement made above for the head characters.

TABLE V.

Coefficients of Correlation.

Character	*L.M.F.*	*L.F.*	*L.C.*	*S.*
L.M.F.	1·000	0·763	0·832	0·661
L.F.	0·763	1·000	0·757	0·715
L.C.	0·832	0·757	1·000	0·803
S.	0·661	0·715	0·803	1·000

TABLE VI.

Probable Errors of Coefficients of Correlation.

Character	*L.M.F.*	*L.F.*	*L.C.*	*S.*
L.M.F.	0·000	±0·010	±0·007	±0·013
L.F.	±0·010	0·000	±0·010	±0·012
L.C.	±0·007	±0·010	0·000	±0·008
S.	±0·013	±0·012	±0·008	0·000

7. *Regression Equations.* Considering the significancy of all the coefficients of correlation shown above, we may conclude that the variability between the human organs is not chaotic, but governed by very definite laws. We are therefore able to predict an unknown measurement in terms of the given length of a known organ.

Table VII results from this investigation. This table shows us how and with what accuracy human characters can be calculated one from another. The

TABLE VII.

Table for Application of linear Regression Equations.

Calculated measurements	Constants	Computed from			
		L.M.F. in mm.	*L.F.* in mm.	*L.C.* in mm.	*S.* in mm.
L.M.F. in mm.	*m*	—	0·3790	0·2409	0·0684
	c	—	+16·4	+3·9	+0·7
	p.e.	—	±2·7	±2·3	±3·1
L.F. in mm.	*m*	1·5359	—	0·4411	0·1490
	c	+82·4	—	+55·8	+10·6
	p.e.	± 5·4	—	± 5·5	± 5·8
L.C. in mm.	*m*	2·8741	1·2990	—	0·2872
	c	+129·6	+122·8	—	−18·7
	p.e.	± 7·9	± 9·4	—	± 8·5
S. in mm.	*m*	6·3839	3·4304	2·2451	—
	c	+929·9	+774·4	+631·0	—
	p.e.	±29·5	±27·5	±23·5	—

regression equations given in this table are of course linear, and are to be put into the form

$$y = mx + c,$$

where y is the measurement looked for,

x is the known measurement,

m is the change of the dependent measurement for a unit change of the independent variable,

p.e. is the probable error for each calculated measurement, and is computed from

$$0{\cdot}6745\sigma_y n^{\frac{1}{2}}(1-r^2)^{\frac{1}{2}}/(n-2)^{\frac{1}{2}},$$

and c is a constant.

All characters, stature included, are in millimetres.

Besides the formulae given above, the following multiple regression equations for the determination of the stature may be of interest:

Reconstruction of stature from a knowledge of one measurement:

$$\text{Stature} = 6{\cdot}3839\ (\text{Finger}) + 929{\cdot}9,$$
$$\text{Stature} = 3{\cdot}4304\ (\text{Foot}) + 774{\cdot}4,$$
$$\text{Stature} = 2{\cdot}2451\ (\text{Cubit}) + 631{\cdot}0.$$

Reconstruction of stature from a knowledge of two measurements:

$$\text{Stature} = -0{\cdot}2228\ (\text{Finger}) + 2{\cdot}2981\ (\text{Cubit}) + 632{\cdot}2,$$
$$\text{Stature} = 2{\cdot}6699\ (\text{Finger}) + 2{\cdot}4195\ (\text{Foot}) + 730{\cdot}2,$$
$$\text{Stature} = 1{\cdot}7136\ (\text{Foot}) + 1{\cdot}2034\ (\text{Cubit}) + 564{\cdot}2.$$

Reconstruction of stature from a knowledge of three measurements:

$$\text{Stature} = 1{\cdot}4369\ (\text{Foot}) + 1{\cdot}9866\ (\text{Cubit}) - 1{\cdot}0166\ (\text{Finger}) + 496{\cdot}6,$$

where all the values, stature included, are in millimetres.

8. *Summary of Conclusions.*

(*a*) The raw material, though taken from the anthropometrical slips of prisoners, is a fair sample of the ordinary native community.

(*b*) The natives dealt with are not of the poorest nor of the lowest class of the Cairo natives.

(*c*) The records are homogeneous, and are sufficient in number for a good statistical study.

(*d*) The distributions of stature, cubit, finger, etc., are normal.

(*e*) The coefficients of correlation between all organs are significant, no matter to what race the records belong.

(*f*) The variability between the human organs is not chaotic, but governed by definite laws which enable us to predict measurements one from another.

(*g*) The coefficients of correlation for the human organs are practically identical from one race to another, only the coefficients resulting from the head measurements vary to any great extent.

(*h*) The great variation which the coefficient of correlation for the head measurements undergoes from race to race, is to be traced to special factors not yet determined, which influence this measure of relationship. I hope to investigate this further.

TABLE VIII. *Correlation Table of Length and Breadth of Head.*

Length of Head in mm.

Breadth of Head in mm. — Groups	173—175	176—178	179—181	182—184	185—187	188—190	191—193	194—196	197—199	200—202	203—205	206—208	Total
130—132	—	1	—	1	1	1	1	—	1	—	—	—	6
133—135	—	—	—	2	2	2	2	—	—	—	—	—	8
136—138	3	3	10	9	9	10	15	8	8	—	—	—	75
139—141	—	1	5	14	17	29	35	8	4	4	3	—	120
142—144	—	2	9	16	24	54	31	24	19	8	1	2	190
145—147	1	1	9	17	39	40	41	29	12	11	1	1	202
148—150	—	—	3	7	19	32	22	13	18	8	1	2	125
151—153	—	—	5	1	2	6	11	13	7	5	1	4	55
154—156	—	—	—	2	1	1	2	2	2	1	—	1	12
157—159	—	—	—	—	—	2	—	1	4	1	—	—	8
160—162	—		—	—	—	—	—	—	—	1	—	—	1
Total	4	8	41	69	114	177	160	98	75	39	7	10	802

TABLE IX. *Correlation Table of Left Foot with Left Middle Finger.*

Left Foot in mm.

Left Middle Finger in mm. — Groups	222—228	229—235	236—242	243—249	250—256	257—263	264—270	271—277	278—284	285—291	292—298	Total
93—96	—	—	—	1	—	—	—	—	—	—	—	1
97—100	1	3	2	—	—	—	—	—	—	—	—	6
101—104	1	5	14	11	1	—	—	—	—	—	—	32
105—108	3	9	28	31	32	7	1	—	—	—	—	111
109—112	—	2	17	42	62	36	15	2	—	—	—	176
113—116	—	—	9	22	63	67	36	14	4	—	—	215
117—120	—	—	—	4	13	46	34	35	2	3	—	137
121—124	—	—	—	—	7	9	27	24	8	4	1	80
125—128	—	—	—	—	—	2	9	12	11	—	2	36
129—132	—	—	—	—	—	—	—	1	—	4	—	5
133—136	—	—	—	—	—	—	—	—	3	—	—	3
Total	5	19	70	111	178	167	122	88	28	11	3	802

TABLE X. *Correlation Table of Left Cubit with Left Middle Finger.*

Left Cubit in mm.

Left Middle Finger in mm. — Groups	388—402	403—417	418—432	433—447	448—462	463—477	478—492	493—507	508—522	523—537	538—552	Total
93—96	—	—	1	—	—	—	—	—	—	—	—	1
97—100	2	1	1	1	1	—	—	—	—	—	—	6
101—104	1	9	17	4	1	—	—	—	—	—	—	32
105—108	—	6	33	59	13	—	—	—	—	—	—	111
109—112	—	4	18	62	75	11	6	—	—	—	—	176
113—116	—	—	1	29	100	68	17	—	—	—	—	215
117—120	—	—	1	5	30	75	21	5	—	—	—	137
121—124	—	—	—	—	4	27	36	11	2	—	—	80
125—128	—	—	—	—	—	5	19	8	4	—	—	36
129—132	—	—	—	—	—	—	1	2	2	—	—	5
133—136	—	—	—	—	—	—	—	—	2	—	1	3
Total	3	20	72	160	224	186	100	26	10	0	1	802

TABLE XI.

Correlation Table of Stature with Left Middle Finger.

Stature in cm.

Left Middle Finger in mm. Groups	*148—151*	*152—155*	*156—159*	*160—163*	*164—167*	*168—171*	*172—175*	*176—179*	*180—183*	*184—187*	Total
93—96	—	—	—	1	—	—	—	—	—	—	1
97—100	2	1	2	—	1	—	—	—	—	—	6
101—104	1	3	17	9	2	—	—	—	—	—	32
105—108	—	11	24	41	24	10	1	—	—	—	111
109—112	1	9	21	64	49	29	3	—	—	—	176
113—116	—	1	9	50	78	51	19	7	—	—	215
117—120	—	3	—	15	43	43	22	7	4	—	137
121—124	—	—	—	5	11	26	25	8	5	—	80
125—128	—	—	—	—	4	6	16	8	2	—	36
129—132	—	—	—	—	—	1	—	1	3	—	5
133—136	—	—	—	—	—	1	1	—	—	1	3
Total	4	28	73	185	212	167	87	31	14	1	802

TABLE XII.

Correlation Table of Left Foot with Left Cubit.

Left Foot in mm.

Left Cubit in mm. Groups	*222—228*	*229—235*	*236—242*	*243—249*	*250—256*	*257—263*	*264—270*	*271—277*	*278—287*	*288—291*	*292—298*	Total
388—402	1	2	—	—	—	—	—	—	—	—	—	3
403—417	—	4	10	6	—	—	—	—	—	—	—	20
418—432	4	8	20	22	16	2	—	—	—	—	—	72
433—447	—	5	24	43	56	28	3	—	1	—	—	160
448—462	—	—	13	32	68	70	32	7	2	—	—	224
463—477	—	—	3	8	28	47	61	37	1	1	—	186
478—492	—	—	—	—	9	19	19	33	17	3	—	100
493—507	—	—	—	—	1	—	5	7	5	5	3	26
508—522	—	—	—	—	—	1	2	4	1	2	—	10
523—537	—	—	—	—	—	—	—	—	—	—	—	0
538—552	—	—	—	—	—	—	—	—	1	—	—	1
Total	5	19	70	111	178	167	122	88	28	11	3	802

TABLE XIII.

Correlation Table of Stature with Left Foot.

Stature in cm.

Left Foot in mm. / Groups	148—151	152—155	156—159	160—163	164—167	168—171	172—175	176—179	180—183	184—187	Total
222—228	1	1	1	2	—	—	—	—	—	—	5
229—235	2	3	8	6	—	—	—	—	—	—	19
236—242	—	10	23	24	10	3	—	—	—	—	70
243—249	1	9	16	41	35	8	—	1	—	—	111
250—256	—	5	21	65	53	30	4	—	—	—	178
257—263	—	—	4	36	57	51	18	1	—	—	167
264—270	—	—	—	9	37	41	24	10	1	—	122
271—277	—	—	—	1	17	29	22	14	5	—	88
278—284	—	—	—	1	3	4	12	4	3	1	28
285—291	—	—	—	—	—	1	4	1	5	—	11
292—298	—	—	—	—	—	—	3	—	—	—	3
Total	4	28	73	185	212	167	87	31	14	1	802

TABLE XIV.

Correlation Table of Stature with Left Cubit.

Stature in cm.

Left Cubit in mm. / Groups	148—151	152—155	156—159	160—163	164—167	168—171	172—175	176—179	180—183	184—187	Total
388—402	3	—	—	—	—	—	—	—	—	—	3
403—417	1	15	3	1	—	—	—	—	—	—	20
418—432	—	6	33	27	5	1	—	—	—	—	72
433—447	—	4	27	76	44	8	1	—	—	—	160
448—462	—	3	9	60	91	54	7	—	—	—	224
463—477	—	—	1	17	54	77	29	6	2	—	186
478—492	—	—	—	3	17	26	36	16	2	—	100
493—507	—	—	—	1	1	1	12	6	5	—	26
508—522	—	—	—	—	—	—	2	3	5	—	10
523—537	—	—	—	—	—	—	—	—	—	—	0
538—552	—	—	—	—	—	—	—	—	—	1	1
Total	4	28	73	185	212	167	87	31	14	1	802

A STUDY OF THE CRANIA OF THE MORIORI, OR ABORIGINES OF THE CHATHAM ISLANDS, NOW IN THE MUSEUM OF THE ROYAL COLLEGE OF SURGEONS.

By EVELINE Y. THOMSON, Crewdson-Benington Student in Craniometry, Biometric Laboratory, University College, London.

1. *Introductory.* The fact that pure bred Morioris are now very rare if not extinct renders of special value any study of such crania of them as still exist. In papers referred to below Dr Scott* deals with 50 and Dr Duckworth† with 10 Moriori crania. Professor Thane possesses two specimens and there are a few scattered through various British and Continental museums. Hence the 63 crania in the museum of the Royal College of Surgeons almost double the available number. Yet if we pool the present measurements with such of Dr Scott's series as are homogeneous, our results are still too slender to warrant the publication of more than the measurements themselves and their mean values. As one of the best uses of methods of treatment which do not extend beyond the mean the type sections of the Moriori crania have been reconstructed. By most kind permission of Professor Arthur Keith I have been able to study the collection of Moriori skulls in his charge, and he most readily provided facilities for the photographing of a considerable number of the crania. Although the collection has been for a considerable time in the Royal College of Surgeons no complete set of measurements appears to have been published‡. Dr Duckworth in the paper just cited makes some remarks on a few of the abnormal skulls. Forty-four crania were purchased by the Royal College as "from the Chatham Islands" in 1892, i.e. 765^{1} to 765^{44}, six, 765^{45} to 765^{50}, appear to be later additions, while thirteen, 758–760, 760^{1}, 760^{2}, 761–765, seem to have an earlier origin, as well as the three 1015, 1015^{A} and 1016 from the Barnard Davis collection. The very remarkable character of the Moriori skull is not fully accounted for by the outlying character of their island, although anthropologists naturally look to such islands for the survival of primitive types. Chatham

* Contribution to the Osteology of the Aborigines of New Zealand and of the Chatham Islands. *Transactions of New Zealand Institute*, Vol. XXVI. pp. 1—64. Wellington, 1894.

† On a collection of Crania of the Moriori, *Journal of the Anthropological Institute*, Vol. XXX. p. 141 *et seq.* London, 1900.

‡ Eight have certain measurements given in Flower's *Catalogue of the Royal College of Surgeons.* Barnard Davis gives measurements of three others in his *Thesaurus Craniorum.*

Island lies to the east of New Zealand. According to their own traditions the Moriori reached their present home direct from Hawaiiki, so that they are not degraded Maoris but form a special branch of the Polynesian race. Never numerous, they were mostly slaughtered in the year 1838—two years previous to the first British colonisation of New Zealand—by a party of Ngatilawa landed on the island by a whaler. This party enslaved the few survivors. Some years ago the remaining Moriori were stated to be reduced to fifty, and these were probably not of pure race*. There was nothing of special note in their occupations, habits or beliefs. Most of their life was utilised in procuring food, making garments and sleeping mats, or in shaping stone implements in which they showed considerable ingenuity. Fish hooks, fish spears and gaffs in constant use were cleverly shaped out of bone. The dwellings were ordinary conical huts constructed out of fern tree and thatched with toi-toi grass: they were often sufficiently large to accommodate twenty or thirty persons. Some of them were ornamented with rude carvings, but most of these places have been demolished by the Maori, or perished from age, so that no specimens of their art are now obtainable. They lived peaceably, chiefly along the coast, spending their time as a rule in procuring supplies of fish, fern roots, nuts, wild duck, etc. Periodically they put off in rafts or large canoes for the hunting of seal and albatross. They are said to have had a confused notion of good and evil spirits and to have invoked numerous deities prior to any undertaking of importance†. In short, such accounts as we have of them do not seem to differentiate them markedly from the inhabitants of many other Polynesian islands. They stand out differentiated rather by their cranial than by their cultural characters.

2. *Measurements and Methods of Measurement.* The measurements I have taken are similar to those adopted by previous biometric craniologists. They were made in the manner described by Fawcett (*Biometrika*, Vol. I. p. 416) and Macdonell (*Ibid.* Vol. III. p. 200). The measurements are as follows:

C = capacity taken with mustard seed, packing in skull and in measuring glass, not by our usual laboratory method of weighing as no fine chemical balance was available, the work being done away from the Laboratory. In the tables it will be seen that I have given two figures for each capacity. The seed was tightly packed into the skull and then tightly packed into the measuring glass. This is the standard method. It involves the great labour of double packing; the second packing is avoided in our laboratory use of the balance. It occurred to me to be worth while trying loose packing in the measuring glass and correlating the results for tight packing and loose packing with the view of saving the time employed in the second tight packing on another occasion. The loose packing consisted in taking the seed from the skull, gently pouring it into the measuring glass and then giving a slight shake sufficient to obtain a level surface for reading. The

* R. Lydekker, *Living Races of Mankind*, Vol. I. p. 72.

† Extracted from Notes on the Chatham Islands by J. W. Williams, of Waitangi West, Chatham Island, *Journal of the Anthropological Institute*, Vol. XXVII. p. 343. London, 1897.

results are given in the footnote below*; it suffices to remark here that for practical purposes it appears approximate enough if we deduct one-twentieth from the volume of the loosely packed seed in order to obtain the volume of the tightly packed. My corrected results from loose packing are given in brackets in the tables. The means are:

Tight packing, ♂ 1422·07; Loose packing, ♂ 1423·03,
„ „ ♀ 1310·73; „ „ ♀ 1308·47.

The mean difference (regardless of sign) of the results obtained from loose packing from those obtained from tight packing is, however, 8 cm.³ How much of this is due to variation in the tight packing in the measuring glass, how much to fortuitous settlements in the loose packing it is not possible to say. The differences suggest at any rate that a method may well be devised by which tight packing in the measuring cylinder may be avoided.

Further measurements made are:

F = Flower's Ophryo-occipital length. L' = glabellar horizontal length. L = maximum length from glabella to occiput. B = maximum horizontal breadth. B' = least forehead breadth from one temporal crest to the other. H = Basio-bregmatic height. OH = auricular height as measured on the

* Let C_t be the capacity from tight packing, C_l from loose packing; let σ be a standard deviation, V a coefficient of variation and r_{tl} the correlation of C_t and C_l, then in cubic centimetres:

For male crania.		*For female crania.*	
Mean C_t = 1422·07	Mean C_l = 1498·11	Mean C_t = 1310·74	Mean C_l = 1377·37
σ_t = 110·185	σ_l = 113·263	σ_t = 90·265	σ_l = 90·122
V_t = 7·75	V_l = 7·56	V_t = 6·89	V_l = 6·81
r_{tl} = ·9935		r_{tl} = ·9969	

It will be seen that the correlations are very high. The regression lines giving C_t for C_l are:

$$C_t = \cdot 96654\, C_l - 25{\cdot}91 \text{ for males,}$$
$$C_t = \cdot 99847\, C_l - 64{\cdot}53 \text{ for females.}$$

We should have expected the same line to have resulted from the two series, as it does not it is clear that the coefficient of C_l is a function of C_l. To determine the nature of loose packing a little further we took the regression lines of $(C_l - C_t)/C_l$ and C_l or equations of the form $(C_l - C_t)/C_l = AC_l + B$. There resulted:

$$(C_l - C_t)/C_l = \cdot 06843 - \cdot 0000{,}1173\, C_l \text{ for males,}$$
$$(C_l - C_t)/C_l = \cdot 056939 - \cdot 0000{,}0619\, C_l \text{ for females.}$$

These may be written:

$$C_t = C_l\,(\cdot 93157 + \cdot 0000{,}1173\, C_l) \text{ for males,}$$
$$C_t = C_l\,(\cdot 94306 + \cdot 0000{,}0619\, C_l) \text{ for females.}$$

The terms depending on C_l^2 are so small that we may insert in them for C_l, the mean values in each case, when we find:

$$C_t = C_l\,(\cdot 94914) \text{ for males,}$$
$$C_t = C_l\,(\cdot 95159) \text{ for females.}$$

The differences are now not unreasonable considering the paucity of the data, and we see that very approximately it suffices to take for both sexes, $C_t = C_l\,(1 - \frac{1}{20})$, the rule given above.

The whole matter is to be reconsidered on more ample material. It is not clear at present how far it depends on personal equation in tight and loose packing of different amounts and how far on the diameter of the measuring glass, i.e. the pressure varying with height or with C_l and so affecting the looseness of the loose packing.

craniophor, that is the height perpendicular to the horizontal plane above the line joining the upper points of the margins of the auricular orifices. LB = length from basion to nasion. $\angle P$ = profile angle, measured with goniometer, the skull being on the craniophor. Q = transverse arc from upper margin of one auricular passage to upper margin of the other, the arc being taken perpendicular to the horizontal plane with the skull on the craniophor. This perpendicular arc goes generally behind the bregma through a point, the "apex," which is marked on the skull by the sliding bar of the craniophor, i.e. the bar by which OH is obtained. Q' is the same measure taken when the skull is off the craniophor*. S = sagittal arc from nasion to opisthion. S_1 = arc from nasion to bregma, S_2 = arc from bregma to lambda, S_3 = arc from lambda to opisthion. S'_3 = chord from lambda to opisthion. U = horizontal circumference measured directly above the superciliary ridges and round the most projecting part of occiput. $G'H$ = upper face height from nasion to alveolar point. GB = face breadth from the lower end of one zygomatic-maxillary suture to that of the other. J = zygomatic breadth, from the most lateral point of one zygomatic arch to that of the other. NH = nasal height from nasion to the lowest edge of left pyriform aperture (in cases where this is damaged the lowest edge of right aperture is taken; if there be a marked difference between the heights of the two apertures, both heights are given). NB = nasal breadth, greatest breadth of nasal aperture, wherever it may be. O_1 = breadth of orbit, R and L, the greatest breadth from side to side†. O_2 = height of orbit, R and L, taken perpendicular to O_1. G_1 = length of palate, from the point of the spina nasalis posterior to an imaginary surface tangential to the inner alveolar surfaces of the mesial incisors. G'_1 = length from base of spine. G_2 = breadth of palate between the inner alveolar walls at the second molars. GL = profile, or basi-alveolar length, from basion to alveolar point. fml = length of foramen magnum; fmb = greatest breadth of foramen magnum. GH = face height.

Weight of skull in grams. In this series the actual weight of the skull (measured without mandible) was taken, as the crania being so abnormally heavy, it was thought the individual weights of the skulls might be of some interest.

The indices calculated are: the cephalic indices 100 B/L', 100 H/L', 100 B/L, 100 H/L and 100 B/H; the upper facial index 100 $G'H/GB$; the nasal index 100 NB/NH; the right and left orbital indices 100 O_2/O_1; the palate index 100 G_2/G_1; the occipital index $Oc.\ I. = 100\,\frac{S_3}{S'_3}\sqrt{\frac{S_3}{24(S_3-S'_3)}}$; the foraminal index $100\,\frac{fmb}{fml}$. Finally the gnathic index has been obtained from mean values.

* Q is taken with the tape over the skull and measured from the top of one ear rod to the top of the other; it is somewhat troublesome to take. Q' is easier, but owing to the recessing of the margins of the auricular passages is not quite the same as Q. The differences in Q and Q' are partly due to this, partly to observational error. The mean values: Q = 316·11 for ♂s, = 310·00 for ♀s, and Q' = 315·94 for ♂s. = 310·20 for ♀s, show that there is close concordance of both methods in determining type.

† On the difficulties of measurement of O_1: see *Biometrika*, Vol. III. p. 201.

The twelve mandibles available were measured: w_1 = condylar width, or greatest width from outside one condyle to outside of second; w_2 = greatest width at angles from outside one angle to outside of the other; h_1 = greatest height, from lowest median projection to top of alveolar margin between mesial incisors; and zz = distance between foramina mentalia. The mandibles were too few in number to justify a more elaborate system of measurements. Also the following angles were taken in the manner described by Cicely Fawcett*: (*a*) profile angle ($\angle P$) as already mentioned measured with Ranke's goniometer; (*b*) angle between upper face length and profile length, the alveolar angle ($\angle A$), determined from the length measurements GL, $G'H$ and LB by aid of Pearson's trigonometer; (*c*) angle between skull basis, and upper face length, the nasal angle ($\angle N$), determined as in (*b*); (*d*) angle between skull basis and profile length, the basilar angle ($\angle B$), determined as in (*b*); (*e*) angle between skull basis and the standard horizontal plane (θ_1), the basio-nasal horizontal angle, determined as in (*b*) with the use of profile angle as found by goniometer. (*f*) Angle between profile length and the standard horizontal plane (θ_2), the basio-alveolar horizontal angle, determined as in (*e*).

Sex. I first recorded my own opinion as to the sex of the skulls, and then Professor Pearson gave me his judgment on them. In one or two cases, where it was most difficult to determine the sex, and we could not agree, appeal was made to Professor Keith, and he kindly gave his opinion also.

3. *Contours.* A study of the three chief contours of these crania was made. These contours were drawn by aid of a Klaatsch contour tracer, in the same manner as the contours traced by Dr Crewdson Benington in his paper in *Biometrika*, Vol. VIII. p. 123, on Cranial Type Contours. From these contours were obtained the *average* or *type* contours of the Moriori craniæ. The construction of the type contours was as follows:

(*a*) *The transverse, vertical, or auricular coronal section.* The skull being placed on the craniophor, and adjusted in the usual manner to the standard horizontal plane, the horizontal rod of the craniophor determines the "apex" of the skull. The plane through the apex of the skull (which is marked with a pencil stroke) and the "auricular points" is the transverse vertical plane of the skull. The skull being now transferred to the Klaatsch apparatus, the vertical plane is made horizontal and the contour traced from "auricular point" to "auricular point" through the apex. The line joining auricular point to auricular point is the auricular line of the contour, and the line perpendicular to it through its mid-point is the vertical axis MA of the transverse vertical contour. This vertical axis is divided into ten equal parts by aid of the proportional compasses, and lines parallel to the auricular line are drawn through the points of division; these lines will not necessarily be bisected by the vertical axis owing to the asymmetry of the skull. They are numbered from below upwards 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,

* *Biometrika*, Vol. I. p. 418.

where 1 represents the lengths on the auricular axis. An eleventh line is taken one-quarter of the last section from the apex, the distances to *R* and *L* of the vertical axis up to the contour are measured off along these eleven horizontals. Also the length of the vertical axis *MA* is measured. Since the skull is placed with the occiput downwards on the Klaatsch, the right side of the skull is to the observer's left when viewing the contour, and the left of the skull to his right. This is reversed in plotting type-contours in order to obtain the *norma occipitalis**.

The following measurements are now entered in the columns of the record for each contour—the sum of 1*R* and 1*L* being the interauricular diameter:

M.A.	1*R*	1*L*	2*R*	2*L*	3*R*	3*L*	4*R*	4*L*	5*R*	5*L*	6*R*	6*L*

7*R*	7*L*	8*R*	8*L*	9*R*	9*L*	10*R*	10*L*	$A\frac{1}{4}R$	$A\frac{1}{4}L$

The means of the columns give the corresponding lengths for the type skull and from these we can plot the points on the type section; this section is then drawn between the points by aid of a spline, and we thus reach the transverse type contour. See Figs. III and IV for male and female respectively.

(*b*) *The glabellar horizontal section.* This is obtained by placing the skull on the craniophor, and by aid of the scriber marking points on the same horizontal plane as the nasion and the gamma, in the region above both auricular passages. The skull is now adjusted in the Klaatsch holder, apex downwards, until these four points are shown to be in the same horizontal plane. The horizontal contour is then drawn parallel to this plane but through the glabella; it will fall below the lambda, and does not pass far from the occipital point. The points in the median plane immediately above the nasion and below the lambda on this contour are especially marked with the Klaatsch tracer. They will be spoken of as *F* and *O*, the glabellar and occipital points. *FO* is the fronto-occipital line of the horizontal section. This fronto-occipital line being found, it is divided into ten equal parts by the proportional compasses, and lines perpendicular to the fronto-occipital line are drawn through the points of division, and the lengths up to the contour measured right and left. To obtain the frontal and occipital curvatures, parallel lines at $F\frac{1}{4}$ and $O\frac{1}{4}$ were taken as in the other cases.

In the case of the Moriori contour there were such sharp projections at the *linea temporalis* that ordinates perpendicular to the fronto-occipital line both right and left were drawn to the most projecting points. These are the ordinates

* Benington (*Biometrika*, Vol. VIII. p. 151 etc.) gives the transverse section as *norma facialis*, and his tracings must be reversed in superposition on mine.

T_R and T_L with abscissae x_R and x_L. To complete this contour in the neighbourhood of the glabella extra lines were measured at quarter and half the distance from *F* to 2; these lines are spoken of as the $F\frac{1}{4}$ and $F\frac{1}{2}$ lines. All these points being recorded they were found sufficient by aid of a spline to reconstruct the horizontal section effectively.

When we look at any horizontal contour with the frontal point towards us, the right of the skull is on our right, as it would be if we were facing the actual skull the right way up. The ordinates are reversed in plotting the type contour in order to obtain the *norma verticalis*.

We have the following measurements entered in our recording book:

FO	$F\frac{1}{4}R$	$F\frac{1}{4}L$	$F\frac{1}{2}R$	$F\frac{1}{2}L$	2*R*	2*L*	3*R*	3*L*	4*R*	4*L*	5*R*	5*L*	6*R*	6*L*

7*R*	7*L*	8*R*	8*L*	9*R*	9*L*	10*R*	10*L*	$O\frac{1}{4}R$	$O\frac{1}{4}L$	T_R	T_L	x_R	x_L

The means are again found for all the columns. The results are plotted, splined in, and we obtain the type horizontal section for ♂ and ♀. See Figs. V and VI.

(*c*) *The sagittal or median section.* The skull being again placed on the craniophor and adjusted to the Frankfurt horizontal plane, the point γ on the same horizontal level as the nasion is marked off on the median plane of the skull by means of the scriber. The skull is now removed to the Klaatsch, and adjusted so that the nasion, bregma and lambda lie in the same horizontal plane. The contour is then drawn, starting from the alveolar point. The alveolar point, the nasion, the bregma, the apex, the vertex, the lambda, the gamma, the inion, the opisthion, the basion, and the auricular point are all marked on this contour drawing. We now take the horizontal line through the nasion, i.e. the line linking the nasion to the gamma, as our "horizontal axis" and divide it into ten equal pieces by aid of the proportional compasses. The nasion counting as 0, and the other divisions from nasion to gamma as 1, 2, 3, 4...9, ordinates are drawn through these ten points perpendicular to the horizontal axis. Lines 8 and 9 are produced, and continued on the other side of the horizontal axis; a further upright is taken near the gamma at $\frac{1}{4}$ the distance from the gamma to 9; this for brevity is termed the $\gamma\frac{1}{4}$ line; this line is also produced and continued below the horizontal axis. At 0, i.e. vertically through the nasion, there will in most male skulls be a sensible ordinate roughly of about 20 mm. In order to get the curvature in the neighbourhood of the glabella a *vertical* tangent was drawn by aid of the set-square to the most projecting part of the glabella; the point in which the tangent meets the

prolonged gamma-nasion line being T and the point of contact of the tangent being G, the ordinate GT and the abscissa TN represented by the letters G_y and G_x respectively were measured, they are practically the coordinates of the glabella. To complete the drawing of the frontal part of the sagittal contour, an extra ordinate was measured at $\frac{1}{4}$ the distance from 0 to 1; this line is spoken of as the $N\frac{1}{4}$ line. Perpendiculars were also dropped from the opisthion, auricular point and suborbital point to $N\gamma$. Also a perpendicular from the most projecting point of the contour just above the inion was drawn to $N\gamma$. These points will now be found amply sufficient to reproduce the individual contour as closely as it could be redrawn by the Klaatsch.

In our record book, we have accordingly the following entries, as well as the number and sex of the cranium:

$N\gamma$	$N\beta$	β-sub. to $N\lambda$	β-sub. to GI	V-sub. to GI	G_x	G_y	Ordinates above $N\gamma$													
							0	$N\frac{1}{4}$	1	2	3	4	5	6	7	8	9	$\gamma\frac{1}{4}$	$\gamma\frac{1}{2}$	γ

Ordinates below $N\gamma$				Occ.		λ		Suborb.		Aur.		Op.		Lengths					Angles					
8	9	$\gamma\frac{1}{2}$	$\gamma\frac{1}{4}$	x	y	x	y	x from N	y	x from γ	y	x from γ	y	GI	NI	γ to Bas.	N to Bas.	γI	$\phi = \angle \beta GI$	$\phi' = \angle \beta NI$	$A\angle$	$N\angle$	$B\angle$	$P\angle$

The type contours are constructed as before. See Figs. I and II.

(*d*) *General remarks on Contours.* Illustrations of the type contour of the Moriori are shown in Figs. I to VI. For the sagittal contour:

N = Nasion, *Gl.* = Glabellar point, B = basion, β = Bregma, V = vertex, λ = Lambda, γ = Gamma, I = Inion, *Op.* = Opisthion, *Aur.* = Auricular point, *Bas.* = Basion, *Alv.* = Alveolar point, *Suborb.* = Infra-orbital point. *Suborb.* to *Aur.* if joined by a line gives the standard horizontal plane. $N\gamma$ is the horizontal plane through the nasion. The angles N, B and A are the nasal, basilar and alveolar angles of $NBAlv.$ the fundamental triangle. P = the profile angle measured from the Frankfurt horizontal, ϕ = the angle between the glabella-inion and glabella-bregma lines, ϕ' = the angle between the nasion-inion and nasion-bregma lines. There are three interesting subtenses, s, x and y. y is the maximum subtense or height of the calvaria measured from the glabella-inion line, s is the corresponding bregma subtense and x is the bregma subtense to nasio-lambda line.

The most marked characteristics of the Moriori skull are the depressed and retreating forehead, the bulging glabella, and the high sagittal crest. On

looking at Figs. I and II, it will be seen that these points are well demonstrated in the type sagittal contours. The glabella is not of course so pronounced in the female contour, but it is pronounced for a female skull. Figs. III and IV, illustrating the transverse contours of these skulls, show nothing particularly characteristic; the sagittal crest can be seen, but it is more marked further back than the transverse section. Figs. V and VI, showing the horizontal contours, demonstrate very markedly that the Moriori skull is of a low type. There is a very substantial difference between the type horizontal contour of the Moriori skull and that of the English skull illustrated in Dr Crewdson Benington's paper on Cranial Type Contours: see *Biometrika*, Vol. VIII. p. 123. It may also be noted that in these Moriori horizontal contours, there is a decided flattening at the occipital.

Before we discuss the quantitative value of these contours it is well to examine the arithmetical results obtained for the measured characters.

TABLE I. *Mean Values.*

Character	♂	♀	Character	♂	♀
Capacity	1422·07	1310·73	G_1	53·12	50·46
	(1423·03)*	(1308·47)*	G_2	38·85	37·10
Weight	764·27	622·23	GL	100·93	97·36
F	182·00	176·09	fml	35·29	34·41
L'	185·64	177·77	fmb	30·78	30·15
L	186·87	178·60	GH	121·8	108·57
B	141·40	138·41	w_1	123·6	114·0
B'	95·31	94·01	w_2	106·4	101·87
H	135·91	132·81	h_1	33·17	30·27
OH	117·05	115·08	zz	51·48	48·5
LB	105·60	102·31	$Oc\,I$	62·17	64·15
Q	316·11	310·00	B/L'	76·51	77·90
Q'	315·94	310·20	H/L'	73·23	74·76
S	368·77	354·40	B/L	76·07	77·52
S_1	126·03	121·39	H/L	72·78	74·52
S_2	122·40	117·36	B/H	104·53	103·64
S_3	121·10	115·95	$G'H/GB$	74·13	72·09
S'_3	102·34	99·60	NB/NH	43·91	48·22
U	522·79	502·59	O_2/O_1L	84·03	84·30
$G'H$	76·40	70·01	O_2/O_1R	84·35	84·59
GB	103·15	96·63	G_2/G_1	73·20	73·71
J	137·43	128·93	$\frac{fml}{fmb}$	87·43	87·78
NH	57·33	52·53			
NB	25·30	25·27	$N\angle$	64·84	65·67
O_1R	44·61	42·67	$A\angle$	71·75	73·64
O_1L	44·42	42·36	$B\angle$	43·39	40·67
O_2R	37·59	36·01	θ_2	12·88	10·84
O_2L	37·29	35·52	θ_1	30·33	29·83
G'_1	50·10	47·38	$P\angle$	84°·69	84°·50

* Values obtained by "loose packing."

4. *Mean Values of Characters.* To judge of the degree of accuracy possible in reaching a Moriori type, we must note that the means were based for males on 33 to 35 crania and for females on 20 to 23 crania, according to the character measured; the one exception being capacity, for which only 28 male and 19 female crania were available. The non-adult crania have been omitted from the mean value determinations. The values are given in Table I. The question now arises: how far can we consider these values as adequately describing the Moriori skull? We may first compare our results with those obtained by Dr Duckworth for the Moriori crania at Cambridge*. Owing to the smallness of his series (7 male and 3 female) he has pooled his sexes to obtain his means. We have pooled ours in the same proportions; the following values result:

	Thomson	Duckworth
Maximum Length	184·4	182·5
Maximum Breadth	140·5	142·5
Ophryo-Iniac Length ...	170·5†	173·0
Basi-bregmatic Height ...	135·0	132·5
Nasal Height	55·9	54·2
Nasal Width	25·3	24·2
Basi-nasal Length	104·6	101·2
Basi-alveolar Length	99·9	97·2
Horizontal Circumference ...	516·7	513·6

With the exception of the breadth and our doubtful ophryo-iniac length, Dr Duckworth's crania appear slightly smaller than those at the Royal College of Surgeons. A somewhat closer result is obtained if we pool our males and females without regard to Duckworth's proportions. His numbers are, however, too few for us to do more than say that his results agree not unreasonably with ours‡.

* *loc. cit.* p. 150. No. 4 is given ?♀. No. 2 is given as sex doubtful on p. 142, but as definite ♂ in table on p. 150.

† Obtained only from contours.

‡ The German anthropological catalogue (*Die Anthropologische Sammlungen Deutschlands*, Bd. I. 1877—1902, Berlin v. S. 51—2, Bresike Catalogue, 1880) provides measurements of three Moriori crania from the Chatham Islands in the Anatomical Museum of Berlin University. Two of the skulls are of children and the third of an aged person. They are thus of little use for comparative purposes. The measurements given of one of the young skulls which retains one milk tooth and is metopic are:

$$C = 1300,\ L = 170,\ B = 137,\ H = 126,\ S_1 = 121,\ S_2 = 124,\ S_3 = 108,\ GH = 97.$$

The second child skull is metopic, has no 3rd molars and the palate is not yet closed; further it is deformed, a large area on right side of temporal and left side of parietal being much depressed. The measurements given are:

$$C = 1140,\ L = 164,\ B = 133,\ H = 126,\ S_1 = 116,\ S_2 = 104,\ S_3 = 125,\ GH = 93.$$

The aged skull has depression of the parietals, alveolar rim much absorbed after loss of incisors and the three sutures completely synostosed. The measurements given are:

$$C = 1450,\ L = 183,\ B = 136,\ H = 136,\ S_1 = 130,\ S_2 = 125,\ S_3 = 120.$$

The greatest length, assuming it a male, is somewhat below the average, H is about normal, S_3 and B somewhat below average, but the other characters including the capacity above the average, but not greatly. It appears from the description—namely roof shaped, with slightly retreating forehead,

More valuable comparative material is that of Scott*. I have placed in parallel columns in Table II those measurements of Dr Scott which in any degree correspond with mine. We agree so closely in the main straightforward measurements, that where a really substantial difference is apparent, I think it must be attributable to differences in method of measuring rather than to differences of material. I ought to say that I am responsible for the determination of the means from Dr Scott's tables of measurements. While the sex ratio in my material is roughly as 3 : 2 that of Dr Scott's is 3 : 1, so that he has only about half my number of females, although his males are about the same in number as mine. I think it probable that we have followed a different appreciation of sex, because, while his male measurements are on the whole very close to mine, practically all his absolute female measurements are less; thus, the difference he gets between the sexes is far greater than my difference, and rather greater than is usual among uncivilised races, where there is a greater tendency to equality in the measurements than among civilised races. I would venture with all deference to suggest that possibly size has weighed too much with him in sexing†. The first difference between our results to be noted is 33 cm.^{3} in the male capacity, but he used shot and I used mustard seed and a difference of this order may well arise with such difference of method. It is more or less confirmed by the result for female crania, because although his skulls are distinctly less than mine in size, he gets only a difference of 19 cm.^{3} in the capacity. Our *F*, *L*, *B*, *B'*, *H* and (?) *LB*‡ values are in good agreement for males; so also are our *U*, *G'H*, *J*, *NH*, *NB*, O_2L, G_1, *GL*, *fml*, *fmb*, and all the indices except the orbital index for the same sex. Our difference in the orbital index is due to measurement of the breadth O_1L, but this is precisely a measurement, which, without *absolutely* identical method, often leads to great divergence. Turning to our differences we first note *Q*, the transverse arc; the considerable difference here is repeated in the females.

prognathous, and with the teeth very much ground down—to have the usual Moriori characteristics. Four crania, one a child's, two male and one female, were collected during the *Challenger* Expedition. They are described and measured in the *Challenger Reports, Zoology*, Vol. x. p. 73, Report on the Human Crania. The two males are clearly fine crania, exceeding our average, the female is below average. Three crania are described by Zuckerkandl in the *Reise der Novara*; *Anthropologischer Theil*, 1 Abth. S. 104; but the few measurements given are of small use for comparative purposes. Finally five at Paris are considered by de Quatrefages and Hamy in the *Crania Ethnica*, p. 461, Paris, 1882. The three male crania appear to be very large as compared with our series, giving a mean $C = 1600$ cm.^{3} (!), $U = 548$, $L = 194$, $B = 146$, $H = 137$ and $J = 142$, while the two females have a mean $C = 1565$ cm.^{3} (!), $U = 524$, $L = 185$, $B = 144$, $H = 141$, $J = 132$.

Either the measurements are not according to our standards or the crania have been selected as specially remarkable skulls.

* *loc. cit.* on our p. 82.

† We have included Scott's three doubtful crania with his females, not only because he himself in his table puts them in that section, but because, being the rather larger crania of the section, they somewhat reduce the differences between his and our female mean characters.

‡ I think it possible that Dr Scott and I have not determined the alveolar point in precisely the same manner, and I have accordingly bracketed this measurement and the resulting Gnathic Index in Table II.

TABLE II*.

Mean Characters of Moriori Crania, Thomson and Scott.

	Males			Females		
	Thomson	Scott	Combined	Thomson	Scott	Combined
Capacity ...	1422·07 (28)	1454·97 (29)	1438·81 (57)	1310·73 (19)	1291·22 (9)	1304·46 (28)
F	182·00 (35)	181·08 (32)	181·56 (67)	176·09 (22)	172·60 (10)	175·00 (32)
L	186·87 (35)	186·84 (31)	186·86 (66)	178·60 (22)	175·60 (10)	177·66 (32)
B	141·40 (34)	142·10 (31)	141·74 (65)	138·41 (22)	134·20 (10)	137·09 (32)
B′	95·31 (35)	94·09 (32)	94·73 (67)	94·02 (22)	91·20 (10)	93·14 (32)
H	135·91 (34)	135·59 (32)	135·75 (66)	132·81 (21)	128·10 (10)	131·29 (31)
LB... ...	105·60 (34)	103·75 (32)	[104·70] (66)	102·31 (21)	98·70 (10)	[101·15] (31)
Q	316·11 (34)	307·12 (32)	[310·81] (66)	310·00 (21)	289·00 (10)	[303·23] (31)
S_1	126·03 (34)	130·25 (32)	[128·09] (66)	121·39 (23)	120·40 (10)	121·09 (33)
S_2	122·40 (34)	122·56 (32)	122·48 (66)	117·36 (23)	117·00 (10)	117·25 (33)
S_3	121·10 (34)	122·84 (32)	121·94 (66)	115·95 (22)	114·00 (10)	115·34 (32)
S′	368·77 (34)	375·75 (32)	[372·15] (66)	354·40 (22)	351·40 (10)	353·46 (32)
U	522·79 (34)	521·97 (31)	522·40 (65)	502·59 (22)	493·80 (10)	499·74 (32)
G′H ...	76·40 (35)	75·86 (28)	76·16 (63)	70·01 (21)	67·25 (8)	69·25 (29)
J	137·43 (34)	138·89 (28)	138·09 (62)	128·93 (20)	127·10 (10)	128 32 (30)
NH ...	57·33 (35)	57·50 (32)	57·41 (67)	52·53 (21)	50·90 (10)	52·00 (31)
NB ...	25·30 (35)	26·47 (32)	25·86 (67)	25·27 (21)	24·80 (10)	25·12 (31)
O_1L ...	44·42 (35)	41·53 (32)	[43·04] (67)	42·36 (21)	39·10 (10)	[41·31] (31)
O_2L ...	37·29 (34)	36·75 (32)	37·03 (66)	35·52 (21)	35·30 (10)	35·45 (31)
G_1	53·12 (33)	54·10 (30)	53·59 (63)	50·46 (21)	49·00 (10)	49·99 (31)
GL ...	100·93 (34)	101·42 (31)	101·16 (65)	97·36 (21)	96·50 (10)	97·08 (31)
fml... ...	35·29 (33)	35·53 (32)	35·41 (65)	34·41 (20)	34·00 (10)	34·27 (30)
fmb ...	30·78 (34)	30·94 (32)	30·86 (66)	30·15 (22)	29·70 (10)	30·01 (32)
B/*L* ...	76·07 (34)	76·29 (30)	76·17 (64)	77·52 (22)	76·44 (10)	77·18 (32)
H/*L* ...	72·78 (34)	72·60 (30)	72·70 (64)	74·52 (21)	72·96 (10)	74·02 (31)
NB/*NH* ...	43·92 (34)	46·14 (32)	44·99 (66)	48·22 (21)	48·75 (10)	48·39 (31)
O_2/O_1 ...	84·03 (33)	88·64 (32)	[87·65] (65)	84·30 (21)	90·32 (10)	[86·24] (31)
fml/*fmb* ...	87·43 (32)	87·20 (32)	87·32 (64)	87·78 (20)	87·44 (10)	87·67 (30)
Gnathic Index	95·58† (34)	97·81 (31)	[96·64] (65)	95·16† (21)	97·18 (10)	[95·81] (31)

* Figures in round brackets give the number of skulls measured. Means in square brackets are based on possibly non-comparable methods of measurement. † Found from mean lengths only.

I think it is due to difference in method. Assuming Scott to have measured from auricular margin to auricular margin, he has probably measured the least (or geodesic) arc, and not through the "apex"*. Our difference in the frontal part of the sagittal arc, S_1, is very possibly due to his more careful pressing of the tape round the glabella, especially as it is less sensible in the female results. The upper face height is a measurement subject to some variation and doubt if the alveolar ridge be at all worn; our value of *G′H* seems reasonable, but Scott's to be in excess of any comparative material. His values for the female

* He measured the entire transverse circumference and divided it into "supra-" and "infra-" auricular parts, not further defined.

differ in upper face height in exactly the opposite direction. Thus in the 27 measurements compared in the case of the male crania, our differences in four, the capacity, the transverse arc, the frontal section of sagittal arc and the orbital breadth (involving a fifth, the orbital index) are, I believe, due to differences in the methods of measurement. In the case of the female measurements the comparison is not so satisfactory; not only are my numbers small, but Scott's are still smaller. This fact, however, would not account for his absolute lengths being almost invariably smaller than mine. As I have said I think he may have laid too great stress on mere smallness of the skull in sexing. Notwithstanding the combined measurements, transverse arc, orbital breadth and orbital index excepted, give even in the case of the female crania by no means unsatisfactory results. Unless —which seems now very improbable—further long series of Moriori crania are found, Table II, combining Scott's results with my new measurements, must stand as the best available system of Moriori cranial data. It involves measurements of nearly 100 skulls of a race now practically extinct, and should be helpful in determining the racial affinities of the Moriori.

Before leaving Scott's important paper, I should like to note some of the peculiarities of the Moriori crania emphasised by him. He states that typical Moriori crania are usually large and heavy; our table of comparative measurements (Table III) amply verifies this statement as to size. As to the weights we find:

Mean Weight of Crania.

	Moriori (Thomson)	Fuegian (Sergi and Martin)	Egyptian (Davin)
Males	764·3 grs.	872·3 grs.	623·6 grs.
Females	622·2 grs.	722·5 grs.	482·4 grs.

It is possible that the Fuegian crania were weighed, unlike ours, with mandible attached. If so this would account for some, if not all the difference observed. Clearly the Fuegians have very heavy crania, and the Moriori heavy crania, but the latter are not absolutely unique in this respect. The average weight of an Egyptian mandible is 73·1 grs., but of course in the case of these crania the material is in a very different state to the Moriori crania. Still I do not think that the observed difference, 108 in males and 100 in females, can be wholly due to mandibles being included in Sergi and Martin's weighings of Fuegian and not included in our weighings of Moriori. I am inclined to think that while the Moriori crania are heavy, they are not as heavy as the Fuegian. The contours also bring out the same point of great size.

Scott further draws attention to the prominent parietal eminences, the roof-like vertices and the pentagonal character of the *norma occipitalis*. Our Plates XIV, XV, IV, and VI will amply demonstrate to the reader these Moriori features. Further he remarks on the low flattened retreating frontal region as a most striking feature (cf. our Plates III, VII and XV). This character is well measured

by the *frontal index* = 100 × subtense of nasio-bregmatic arc/nasio-bregmatic chord. The value of this index for the Moriori crania is ♂ 19·39, ♀ 21·14. Comparative numbers are:

Cranium of the Murderer Thurtell	♂ 21·65
Whitechapel English of 17th century	♂ 22·21
Guanche Crania	♂ 22·50
Egyptians of XXVI–XXX Dynasty	♂ 22·77
Eskimo Crania	♂ 22·78
Negroes (Congo)	♂ 22·80
Cro-Magnon Skull	♂ 24·27

It will be seen that the male Moriori stands lowest on the list of racial values available.

Of further points noted by Scott the statement* that "the excess of width over height is generally well-marked; indeed in the most typical members of the group the brain case is distinctly flattened," needs, perhaps, further consideration. The feature in question may be measured by the use of Pearson's index $100\,(B - H)/L$. We have then the following racial table:

Values of $100\,(B - H)/L$.

Würtemberg Women ...	+ 9·6	Aino Women	+ 0·9
Würtemberg Men	+ 9·5	Aino Men	+ 0·9
Bavarian Women	+ 9·2	Gaboon Men (1880)	+ 0·9
Bavarian Men	+ 9·0	Maori Men	+ 0·7
English Women (Moorfields) ...	+ 7·9	Maori Women	+ 0·5
French Men	+ 7·2	Coptic Women	+ 0·5
English Men (Moorfields) ...	+ 7·1	Gaboon Women (1864) ...	+ 0·5
English Women (Whitechapel)	+ 5·6	Theban Mummies, Men ...	+ 0·4
English Men (Whitechapel) ...	+ 4·2	Gaboon Men (1864)	+ 0·0
Moriori Men	+ 3·5	Kaffir Men	+ 0·0
Moriori Women	+ 3·2	Australian Men	− 0·2
Congo Men	+ 2·6	Coptic Men	− 0·2
Australian Women	+ 2·4	Prehistoric Egyptian (Naqada) Men	− 0·3
Long Barrow Women ...	+ 2·3	Gaboon Women (1880) ...	− 0·4
Long Barrow Men	+ 2·2	Zulu Men	− 0·7
Fuegian Men	+ 2·0	Angoni Men	− 1·2
Congo Women†	+ 1·9	Negro Men (North Africa) ...	− 1·3
Theban Mummies, Women ...	+ 1·8		
Prehistoric Egyptian (Naqada) Women	+ 1·0		

Judged by this index the Moriori stand for excess of breadth over height per unit of length at the top of all the early and primitive races and nearest to modern Europeans, and this is a feature which differentiates them from Maori and

* *loc. cit.* p. 25.

† Erroneously placed as − 1·9 in *Biometrika*, Vol. VIII. p. 307.

Australians and markedly from the full-blooded negro races. Scott continues: "the orbits are as a rule high, and the appearance of height is increased by the form of the superciliary ridges; while the nasal opening is narrow*." Table III (p. 98) will show that the orbits (O_2L) are higher than any of the comparative material provided, although the Fuegians come fairly close and the Maori follow them, but they are distinctly higher than modern European races and the Negroes. On the other hand the orbits are distinctly broader than those of any other race, and the result is that relatively to the breadth they about equal the Fuegians in orbit, fall behind the Maori and about equal the Negroes and Bavarians. It would probably be better to state that Moriori orbits are large rather than high. Again the statement that the Moriori nasal opening is narrow requires qualification. Its absolute breadth is about the same as Maori, Fuegian and Aino, and in excess of modern Europeans. It will be found that it is the nasal height of the Moriori which is the noteworthy absolute feature of their noses, and the result is that their nasal index falls below that of any of the races placed in Table III for comparison. This is markedly true of the males.

Of general descriptive properties Dr Scott says there were no metopic crania in his series, that the coronal and sagittal sutures were simple, but the lambdoid often very complicated; these remarks would apply well to the Royal College of Surgeons' collection, although two of the skulls of Moriori children at Berlin are, as might well be, metopic. The Moriori cranium does, indeed, strike one as "primitive," but in a different sense to the Australian cranium. A comparison of the following table will show how very little resemblance there is between the Australian and Moriori crania:

Character	Australians (Robertson†)		Moriori (Thomson)	
	♂ (78)	♀ (22)	♂ (34)	♀ (22)
Maximum Length (L) ...	183·56	175·59	186·87	178·60
Maximum Breadth (B)...	130·60	128·73	141·40	138·41
Basio-apical Height (H)	131·13	124·50	135·91	132·81
100 B/L	71·29	73·36	76·07	77·52
100 H/L	71·50	70·95	72·78	74·52
100 H/B	100·50	96·64	96·12	95·95

It is clear that the Moriori is a much larger skull than the aboriginal Australian, and of wholly different proportions. And yet those who have handled the Moriori skull cannot doubt that in its special manner it is as primitive as the Australian. The very bulging glabella, the low receding forehead, the considerable flattening

* *loc. cit.* p. 25. It should be noted that Scott means these remarks to apply to his first type, but he admits the continuity of his first with his second type (p. 26).

† *Proceedings R. S. Edinburgh*, 1910—1911, Vol. XXXI. pp. 1—16. Craniological Observations on the Length, Breadth and Height of 100 Australian Aboriginal Crania.

on either side of the sagittal suture giving rise to the sagittal crest, combined with great weight strike one at once. Anatomical peculiarities as will be noticed from my "Remarks" are frequent. Thus flattening of the obelion, parietal bulging, ossicles of the lambda are common; epipteric ossicles were noted in 17 out of the 63 crania. There were four cases of asymmetric foramina of which the most pronounced is photographed in Plate XIII. Marked inions or inionic ridges occur in several cases, while of rarer anomalies I noted an *incisura crotaphitico-buccinatorus* in one female, an ossicle of the coronal suture (Plate XV), and para-occipital processes articulating with the axis in other cases. Of non-congenital deformities, perhaps the most remarkable is a healed fracture of the mandible on the left side (Plates XX—XXII). Extreme attrition of teeth* was also noticed in a large proportion of the crania in this collection, a feature on which Dr Duckworth has already laid great stress in his paper. The general asymmetry of the skulls will be well realised, not only from a perusal of the remarks on the individual crania, but also by examining the measurements right and left given on the horizontal and transverse type contours. To sum up in the language of Flower and Turner, the Moriori are a megacephalic, mesatricephalic, metriocephalic, mesoseme (although close to the borders of microseme), leptorhine in male, mesorhine in female, orthognathous, brachyuranic† race. They are thus mediocre in most cranial characters, but their non-mediocre features, those of megacephaly, orthognathy and brachyurany, associate them rather with the more highly civilised than with the low races. As we have already said, the "primitive" character of the Moriori cranium does not lie in the same characters as differentiate the Australian aborigines or even the Negro from the European. It is a type of primitiveness hardly described by such characters as we have studied above. It is, perhaps, better considered in reference to the contours.

* A similar attrition of the teeth was noticed in the prehistoric Egyptians and has been attributed by some to the use of stone mortars for grinding the grain.

† Scott (*loc. cit.* p. 4) defines palato-maxillary index as 100 × palato-maxillary length/palato-maxillary breadth. But his lengths are always less than his breadths, just as are Turner's corresponding values for the four Moriori skulls acquired on the *Challenger* Expedition. Scott has accordingly, since his indices are over 100, really used, like Turner, the ratio of breadth to length. Turner following Flower measures the length as the distance from the alveolar point to the line drawn between the most posterior parts of the maxillary tuberosities. This is not exactly our palate length, but appears close to it in value. On the other hand Scott and Turner include in their palato breadth the alveolar arch and thus get a breadth very largely in excess of ours, and, we consider, no true palate breadth at all. To allow for the breadth of the alveolar arch, we have supposed it proportional to the palate breadth as measured in our manner. But we can only compare our breadth with Scott's breadth for our two series as we have not the two measurements in either case. There results for the mean breadth of the (double) alveolar arch at the second molars:

Moriori males, 26·47, Moriori females, 22·79.

We thus deduce for our Moriori the palato-maxillary index of Turner as 123 for males and 119 for females corresponding to the brachyuranic group. If we reverse the process, altering the above values in the ratio of Scott's palatal breadth for Maori to his palatal breadth for Moriori, i.e. subtract 26·47 × 62·77/65·32 from 62·77 and 22·79 × 59·40/59·89 from 59·40 for males and females respectively, we obtain the values for the palatal breadths on our plan, i.e. 37·33 and 36·80 recorded in brackets under Maori in Table III, and the corresponding values of the palatal index.

TABLE III. *Comparison of Moriori and other Races in Cranial Characters.*

Character	Moriori		Maori		Aino		Fuegian		Egyptian (Naqadas)		Northern Negroes
	♂ (c. 34)	♀ (c. 22)	♂ (c. 43)	♀ (c. 20)	♂ (c. 88)	♀ (c. 63)	♂ (c. 34)	♀ (c. 24)	♂ (c. 98)	♀ (c. 123)	♂ (c. 39)
C	1498	1378	1476	1288	1462	1308	[1474]	[1290]	1381	1288	1348
L	186·9	178·6	185·5	175·6	185·8	177·2	192·0	185·0	185·1	177·5	182·9
B	141·4	138·4	140·1	134·8	141·2	136·8	145·0	141·0	134·9	131·5	133·2
B′	95·3	94·0	95·7	91·2	96·2	92·4	97·1	91·0	91·1	88·2	95·9
H	135·9	132·8	137·6	133·0	139·5	135·1	141·0	137·0	135·2	129·5	135·3
OH	117·1	115·1	—	—	119·3	115·0	—	—	115·5	113·1	115·0
LB	105·6	102·3	103·9	100·5	105·4	100·3	{104·5}	{96·3}	99·3	94·9	103·0
P∠	84°·7	84°·5	—	—	82°·0	81°·0	{82°·0	Sergi}	84°·4	84°·5	82°·0
Q	316·1	310·0	307·5	293·8	328·5	317·1	{316·5}	{298·6}	304·2	296·5	306·8
S	368·5	354·4	378·1	361·8	372·8	360·7	{390·0}	{370·0}	373·0	363·6	367·7
U	522·8	502·6	518·0	496·8	522·5	501·7	531·0	502·0	511·0	493·7	508·5
G′H	70·6	69·6	71·4	64·4	69·8	65·5	{71·3}	{69·7}	67·6	65·8	68·0
GB	103·2	96·6	—	—	102·1	96·7	[110·0?]	[103·2?]	95·9	92·5	96·6
J	137·4	128·9	136·8	127·9	137·3	129·7	143·6	132·1	125·6	117·0	129·5
NH	57·3	52·5	53·8	49·4	50·5	47·4	{54·0}	{49·6}	48·9	46·7	49·5
NB	25·3	25·3	25·6	23·4	25·6	24·7	{26·3}	{23·8}	25·1	24·3	27·3
O_1L	44·4	42·4	40·5	39·2	40·9	39·8	{42·5}	{40·3}	38·1	37·2	39·6
O_2L	37·3	35·5	35·0	33·6	34·9	33·9	{36·0}	{34·7}	32·3	32·0	34·2
G_1	53·1	50·5	52·4	48·5	53·0	51·4	[{56·0}]	[{54·0}]	55·8	53·9	49·4
G_2	38·9	37·1	[37·3]	[36·8]	38·2	37·4	[{43·0}]	[{39·0}]	40·3	38·9	39·7
GL	100·9	97·4	100·8	96·9	104·9	100·1	{100·0}	{96·0}	94·7	91·0	105·3
fml	35·3	34·4	35·2	33·1	—	—	{36·0}	{37·3}	—	—	—
fmb ...	30·8	30·2	30·8	29·0	—	—	{31·0}	{31·7}	—	—	—
100 *B/L* ...	76·1	77·5	75·4	74·7	76·5	77·6	76·6	77·7	73·0	74·2	72·9
100 *H/L* ...	72·8	74·5	74·7	74·2	75·6	76·7	73·4	72·9	73·3	73·2	74·2
100 *B/H* ...	104·5	103·6	[101·8]	[101·4]	101·24	101·24	[102·8]	[102·9]	99·8	101·6	98·3
100 *NB/NH* ...	43·9	48·2	47·9	49·1	50·7	50·5	{48·5}	{47·9}	51·1	52·3	55·4
100 O_2/O_1 ...	84·0	84·3	86·1	86·2	85·3	85·2	{84·6}	{85·0}	84·8	86·0	87·2
100 G_2/G_1 ...	74·1	72·1	[71·2]	[75·9]	72·1	72·8	[{76·8}]	[{72·2}]	71·9	72·3	81·5
100 *fmb/fml* ...	87·4	87·8	87·4	88·6	—	—	89·0	84·2	—	—	—
Gnathic Index	[95·6]	[95·2]	97·0	96·4	[99·5]	[99·8]	96·2	99·3	[95·4]	[95·9]	[102·3]

Character	Congo Bantus		Gaboon Bantus		English (Whitechapel)		Long Barrow		Bavarian (Alt-bayerische)	
	♂ (c. 50)	♀ (c. 27)	♂ (c. 50)	♀ (c. 44)	♂ (c. 72)	♀ (c. 80)	♂ (c. 12)	♀ (c. 8)	♂ (c. 100)	♀ (c. 100)
C	1344	1206	1381	1232	1477	1300	—	—	1504	1336
L	177·8	171·4	179·5	171·6	189·1	180·4	190·6	182·6	180·6	173·5
B	138·5	130·9	135·5	130·6	140·7	134·7	142·4	138·6	150·5	144·0
B′	97·5	92·6	96·5	92·8	98·0	93·1	98·9	94·1	103·7	96·3
H	133·8	127·5	135·4	130·1	132·0	124·6	137·8	135·1	133·8	128·0
OH	113·9	109·Q	—	—	114·6	109·2	120·7	118·0	[111·5?]	[109·0?]
LB	98·2	93·3	100·3	95·8	101·6	95·3	102·0	96·8	100·3	96·3
P∠	82°·8	81°·1	—	—	86°·1	87°·1	83°·0	?	89°·1	88°·8
Q	310·7	298·2	310·5	296·3	307·9	294·0	321·8	312·0	329·7	318·7
S	361·8	349·8	364·3	350·4	377·1	362·8	384·8	382·0	365·1	353·4
U	504·3	489·3	501·2	481·2	524·3	503·8	534·9	518·7	524·4	501·4
G′H	63·4	59·3	66·4	62·7	70·2	65·9	69·9	66·8	70·8	66·8
GB	94·8	92·2	95·6	90·9	90·9	84·9	95·9	92·8	95·1	89·7
J	126·5	122·1	129·0	120·5	130·1	120·3	134·0	132·5	135·0	126·3
NH	47·2	45·0	48·2	45·1	51·2	48·7	49·4	47·0	50·9	48·2
NB	26·0	25·0	26·6	24·7	24·3	23·2	24·1	22·8	24·8	23·7
O_1L	40·4	40·5	41·5	39·6	43·1	41·2	38·1	36·9	39·9	38·3
O_2L	34·6	33·4	35·0	33·7	33·5	33·6	31·8	30·6	33·7	33·2
G_1	50·3	50·2	52·1	50·1	48·3	45·1	55·3	57·0	[44·3?]	[42·2?]
G_2	38·9	35·9	39·5	37·4	36·8	35·2	43·7	{36·0}	[33·2?]	[32·1?]
GL	96·5	93·9	100·5	98·5	95·9	90·4	95·3	92·6	—	—
fml	35·9	35·3	35·9	34·0	35·4*	34·3*	35·7	34·5	—	—
fmb	30·0	28·8	30·0	28·1	29·7*	29·0*	27·7	30·3	—	—
100 *B/L* ...	78·0	76·5	75·5	76·2	74·3	74·7	74·9	76·3	83·2	83·1
100 *H/L* ...	75·4	74·6	75·5	75·7	70·0	69·1	72·7	74·0	74·2	73·9
100 *B/H* ...	103·6	102·6	100·1	100·7	106·3	108·5	[103·3]	[102·6]	112·5	112·5
100 *NB/NH* ...	55·2	55·8	55·3	54·9	47·6	47·8	49·0	49·1	48·7	49·2
100 O_2/O_1 ...	85·7	82·5	84·6	85·2	77·9	81·7	83·4	81·7	84·5	86·6
100 G_2/G_1 ...	76·8	70·8	76·1	74·8	76·3	77·7	[79·0]	[{63·2}]	[74·4]	[76·0]
100 *fmb/fml* ...	83·5	81·8	83·9	81·2	84·2*	84·5*	[77·7]	[87·7]	—	—
Gnathic Index	[98·2]	[100·6]	[100·4]	[101·8]	[94·4]	[94·8]	[99·4]	[99·9]	—	—

* Values for English (Moorfields).

5. *Comparison of Moriori cranial Characters with those of other Races.* In Table III I have gathered together such material as seemed of interest for comparative purposes. Unfortunately I have not been able to obtain a good series of Australian measurements for comparison, but the values of the few characters given by Robertson for fairly long series, show that there is little relationship between the two races. In the first place we put Scott's Maori values. We follow these by data for the Aino, the means being calculated from the measurements in Koganei's* Tables; then such slender results as we have been able to obtain for the Fuegians†. Lastly we place three Negro races, North Africans‡ and Benington's Congo and Gaboon measurements§, to complete the series of non-European primitive races. For European races we give English‖ and Bavarians¶ as modern representatives, and the Long Barrow** as an ancient race; unfortunately with very slender numbers. Finally the Egyptians find representation in Fawcett's Naqada measurements††.

In Table III we have been obliged to make various assumptions to deduce certain of the values provided. They are either not provided by the authorities cited, or have been differently measured. The difficulties are greatest when we come to deal with orbital and palate measurements, where most divergence of method has existed. We have in certain cases had to allow for length of spine or breadth of alveolar arch as obtained from other cranial series. Again some authorities do not record the index‡‡ 100 B/H or the gnathic index, and these have had to be supplied from the ratio of mean values. All such measurements are enclosed in square brackets, and when especially doubtful are marked by queries. In other cases where the numbers dealt with are quite inadequate the values are enclosed in curled brackets. We cannot in the present paper deal at length with the basis of these corrections and modifications but a few words as to our treatment of Fuegian data may make clear certain points. The fundamental memoir here is that of Rudolf Martin. He had only five Fuegian crania, but before his date,

* "Beiträge zur physischen Anthropologie der Aino." *Mittheilungen aus der medicinischen Facultät der k. j. Universität.* Tokio, 1894, Bd. II. Reduced in 1900 in the Biometric Laboratory.

† Rudolph Martin, "Zur physischen Anthropologie der Feuerländer," *Zeitschrift für Anthropologie*, Bd. XXII. S. 155—218, 1893. J. V. Hultkrantz, "Zur Osteologie der Ona und Yahgan Indianer des Feuerlandes," *Wissenschaftliche Ergebnisse der schwedischen Expedition nach den Magellanländern 1895—97 unter Leitung von O. Nordenskjoeld.* Stockholm, 1900.

‡ Reduced in the Biometric Laboratory in 1900 from data in the German Anthropological Catalogues.

§ R. Crewdson Benington, "A Study of the Negro skull with special reference to the Congo and Gaboon Crania." *Biometrika*, Vol. VIII. pp. 292—337, 1912.

‖ Results for Whitechapel 17th century English used; see W. R. Macdonell, "A Study of the Variation and Correlation of the Human Skull, with special reference to English Crania." *Biometrika*, Vol. III. pp. 191—244, 1904.

¶ Reduced in Biometric Laboratory in 1900 from the data in J. Ranke's paper, "Beiträge zur physischen Anthropologie der Baiern," Bd. I.

** Edgar H. J. Schuster, "Long Barrow and Round Barrow Skulls in the Oxford Museum." *Biometrika*, Vol. IV. pp. 351—362, 1905.

†† C. D. Fawcett, "A Second Study of the Variation and Correlation of the Human Skull with special reference to the Naqada Crania." *Biometrika*, Vol. I. pp. 408—465, 1902.

‡‡ Unfortunately in the tables (*Biometrika*, Vol. I. pp. 426—7) Fawcett has for the two races Bavarian and Aino (♂ and ♀) the values of 100 H/B put under 100 B/H.

1893, Mantegazza had dealt with 16, Sergi with 14, Garson with 8, Hyades with 8, Sir William Turner with 4 and Quatrefages with 2 skulls. Since his date Hultkrantz has had six additional specimens to measure. Thus there are really between 60 and 70 Fuegian crania certainly available and no doubt if further search were made we might run the value up to nearly 100, and get substantial and important results. Unfortunately some of the writers referred to made very inadequate series of measurements, even on the few crania they had at their command. Martin puts together a good deal of their measurements, but apparently thinks it sufficient to average their averages. This is somewhat dangerous considering the large differences in the numbers they have dealt with. He does not also, we think, sufficiently emphasise the wide divergencies to be expected when means are based on 2 to 10 crania. We have endeavoured to weight the results of the various recorders and such are the numbers given without *curled* brackets in the Fuegian column. The values in curled brackets are based solely on Martin's 2 ♂ and 3 ♀ crania. The statistically trained mind would say that they are worthless as a determination of type. With this statement we should not fail to concur, did not the means provided by Hultkrantz for his six skulls combined accord so closely with our results for male Fuegians as to force us to the conclusion that his crania were practically all males and that by a lucky chance we have reached even with such miserably slender material results closely approximating to a true Fuegian type*.

Character	Table III ♂	Hultkrantz	Character	Table III ♂	Hultkrantz
Capacity	1474	1443	Nasal Height ...	54·0	54·6
Maximum Length ...	192·0	191·5	Nasal Breadth ...	26·3	25·3
Maximum Breadth ...	145·0	146·1	Orbital Breadth ...	42·5	40·0
Basio-bregmatic Height	141·0	140·6	Orbital Height ...	36·0	35·3
Minimum Frontal Breadth	97·1	98·0	Orbital Index ...	84·6	87·4
Profile Angle	82°·0	83°·6	Cephalic Index ...	76·6	76·3
Horizontal Circumference	531·0	531·0	Height Index ...	73·4	72·1
Sagittal Circumference ...	390·0	381·0	Breadth-Height Index	102·8	103·9
Transverse Circumference	316·5	316·1	Foraminal Length ...	36·0	35·3
Bizygomatic Breadth ...	143·6	143·5	Foraminal Breadth ...	31·0	31·0
Profile Length	100·0	106·0	Foraminal Index ...	89·0	87·9

We see how remarkably alike the results are even for such short series, and there is sufficient indication that with uniform methods of measurement the existing Fuegian crania would suffice to provide a quite good typical series of values.

Turning to examine Table III a little more closely, we see that we can dismiss any relation to the Negro race, either in Sudanese or Bantu form. Egyptians and central Europeans show no affinities. It is only with the English or the Long Barrow crania that there is any approach, and the latter are so slender in number

* A more thorough collection and examination of all available Fuegian data will shortly be undertaken.

that it seems idle to deal in more detail with them. On the other hand there is undoubted affinity to the Maori; and rather surprising resemblances to the Aino and to the Fuegians are worth taking into consideration. The full discussion of the proper method of representing racial affinity by a single coefficient must remain undiscussed at present. A rough measure will only be used here; we take the mean percentage difference of each race, for males only, on the basis of the Moriori means, and then consider the mean of these percentages as a working coefficient.

Table IV gives these percentage differences to a single decimal place, and we note at once the following points:

(i) While the Maori are nearest of the races considered to the Moriori, both Aino and Fuegian are surprisingly close.

(ii) Seventeenth century English are more distant than the Fuegians, but have very far from the deviation of a race like the Sudanese Negroes (i.e. 3·92 as compared with 5·80).

(iii) The current view that the Moriori are like the Maori an intermixture of Polynesian and Melanesian stock scarcely receives confirmation from Table IV; the Moriori stand nearer to the Londoner of the 17th century than to the Negro.

(iv) The Maori themselves are still more closely related to the Fuegian (2·87 as against 3·13) than the Moriori.

Thus it is difficult to believe that there has not at some time been a race-link between the present denizens of extreme South America and the denizens of extreme Polynesia.

It is very regrettable that our data for the Long Barrow inhabitants of Britain are so slender. Macdonell has already pointed out their relative closeness to 17th century Londoners and the work of Crewdson Benington* seems to show that the English commonalty of to-day is one with the 17th century Londoners. We have to look to the bulk of the English population to-day as being non-Teutonic in origin, and much talk of our Germanic relationship may ultimately prove to be idle. Craniologically the English commonalty belong in bulk to an earlier and probably more primitive race. It would be unprofitable to speculate until we have larger series and more complete measurements—especially of Polynesian, Melanesian and American Indian crania—on the meaning of the results suggested in the present comparison between the denizens of the extreme fringes of enormous areas. They give one at least a suggestion of vast folk-wanderings in a world possibly of different conformation. Practically they warn us at least to consider the problem of the relationship of the American Indian to either a north-Asiatic or a Polynesian stock as still unsettled†. Even the English relationship shows that the European origin of the American is still a workable hypothesis. We have confidence that if our craniological series consisted of hundreds instead of often units, statistical methods would enable us to construct a racial human

* *Biometrika*, Vol. VIII. p. 131.

† It is in our opinion impossible to accept the views of either Brinton or Hale as final.

pedigree which would throw immense additional light on human descent. The first desideratum at the present time is the collection of crania from what we may call "fringe" races, and the second desideratum the construction of type crania, both by mean measurements and by contours, of the material thus collected. Only then will the science of craniology become a really useful guide to the anthropologist. At present arguments based on half-a-dozen crania—unaccompanied by any previous study of cranial variability—are of the most slender weight and should simply be disregarded by the anthropologist*.

TABLE IV. *Comparison of Male Crania of Moriori with those of other Races.*

Character	Percentage excess value of Moriori character over:				Northern negro
	Maori	Aino	Fuegian	English	
Capacity	+ 1·5	+ 2·4	+ 1·6	+ 1·4	+10·0
Maximum Length ...	+ 0·7	+ 0·6	- 2·7	- 1·2	+ 2·1
Maximum Breadth ...	+ 0·9	+ 0·1	- 2·5	+ 0·5	+ 5·8
Minimum Frontal Breadth	- 0·4	- 0·9	- 1·9	- 2·8	- 0·6
Basio-bregmatic Height ...	- 1·3	- 2·6	- 3·8	+ 2·9	- 0·4
Auricular Height ...	—	- 1·9	—	+ 2·1	+ 1·7
Cranial Base	+1·6	+ 0·2	+ 1·0	+ 3·8	+ 2·5
Profile angle	—	+ 3·2	+ 3·2	- 1·7	+ 3·2
Transverse Arc	+ 2·7	- 3·9	- 0·1	+ 2·6	+ 2·9
Sagittal Arc	- 2·6	- 1·2	- 5·8	- 2·3	+ 2·2
Horizontal Circumference	+ 0·9	+ 0·1	- 1·6	- 0·3	+ 2·7
Upper Face Height ...	- 1·1	+ 1·1	- 1·3	+ 0·6	+ 3·7
Face Breadth	—	+ 1·1	- 6·6	+11·9	+ 6·3
Bizygomatic Breadth ...	+ 0·4	+ 0·1	- 4·5	+ 5·4	+ 5·1
Nasal Height	+ 6·1	+11·9	+ 5·8	+10·6	+13·6
Nasal Breadth	- 1·2	- 1·2	- 4·0	+ 4·0	- 7·9
Orbital Length	+ 8·8	+ 7·8	+ 4·3	+ 2·9	+11·0
Orbital Breadth	+ 6·2	+ 6·4	+ 3·5	+10·2	+ 8·3
Palate Length	+ 1·3	+ 0·2	- 5·5	+ 9·0	+ 6·8
Palate Breadth	+ 3·9	+ 1·8	-10·6	+ 5·4	- 2·3
Profile Length	+ 0·1	- 4·0	+ 0·9	+ 5·0	- 4·4
Foraminal Length ...	+ 0·0	—	- 2·0	- 0·0	—
Foraminal Breadth ...	- 0·0	—	- 0·6	+ 3·6	—
Cephalic Index	+ 0·8	- 0·5	- 0·7	+ 2·4	+ 4·2
Vertical Index	- 2·6	- 3·8	- 0·8	+ 3·8	- 1·9
Breadth-Height Index ...	+ 2·6	+ 3·2	+ 1·5	- 1·6	+ 5·9
Nasal Index	- 9·1	-15·5	-10·5	- 8·2	-26·2
Orbital Index	- 2·5	- 1·5	- 0·7	+ 7·3	+ 3·8
Palatal Index	+ 3·9	+ 2·7	- 3·6	- 3·0	- 9·8
Foraminal Index	+ 0·1	—	- 1·8	+ 3·7	—
Gnathic Index	- 1·5	- 4·1	- 0·6	+ 1·3	+ 7·0
Number of Characters ...	28	28	30	31	28
Mean Percentage Deviation	**2·31**	**3·00**	**3·13**	**3·92**	**5·80**

* It is astounding to note what wide reaching results even the examination of a mere half-dozen Moriori, Fijian, Maori or Eskimo crania led the older craniologists to propound!

6. *Study of the Moriori Type Contours.* The purposes served by these contours are (i) the comparison with the like contours of other races, and (ii) their use as a method for actually determining the mean values of certain characters. We shall consider them from both aspects.

For the first purpose the sole available material consists in the contours for Bantu, Egyptian (XXVI—XXX Dyn.), English, Eskimo and Guanche crania drawn to scale and published with tracings in Crewdson Benington's paper*. Some attempts to obtain type contours by other processes are either based on inadequate numbers, are deduced from far too few plotted points by an unsuitable method, or are reproduced to no standard, or at any rate to no easily comparable scale. For a type contour to be really valuable it should be drawn to full-size and published not only on ordinary but also on transparent paper. It is only in this form that it can be of real value for comparative purposes. Further the contour in the case of the sagittal section should contain the basal triangle and the occipital region. These were included in Dr Benington's individual contours, but unfortunately he omitted to add their mean values to his type contour. We have included them in the Moriori contours.

Starting with the transverse section, we note that the length of the interauricular line is relatively large as compared with the Bantu or Egyptian crania; it is more nearly approached by those of the Guanche or 17th century English, which, however, still fall short of it. The Eskimo transverse section, with a somewhat larger interauricular diameter, has near the "apex" a more marked sagittal crest; this crest so marked in the Moriori crania is, however, thrown as a rule further back in their case. If we consider a primitive form like the Cro-Magnon which has an interauricular length nearly as large as the Moriori type skull, its auricular height is so great that the Moriori transverse section falls entirely inside it. The following table suggests the nature of these results, and confirms the conclusion

Measurements on the Transverse Type Section†, ♂s.

Race	Interauricular Length	Auricular Height	Index
Moriori	125·2	109·5	87·5
Eskimo	128·0	114·2	89·2
English	122·4	111·5	91·1
Guanche	119·6	109·8	91·8
Egyptian	117·4	110·2	93·9
Congo, Bantu‡ ...	116·8	113·6	97·3
Cro-Magnon... ...	124·0	121·0	97·6

* *Biometrika*, Vol. VIII. p. 123.

† These measurements are from the type sections; it must be carefully borne in mind that they may differ from measurements of the same name on the crania themselves.

‡ Fernand Vaz, 1864 Series.

previously reached, i.e. that the Moriori cranium approaches to a certain European type represented by modern English and by prehistoric Guanche* far more closely than it does to the Negro skull.

As measured on the actual skull Scott obtains values of the interauricular length which give a male mean = 126·8, and we have obtained directly for the auricular height of our series of males 117·1; the reasons for these divergencies will be considered later. It would have been of the greatest value had we possessed type sections of the Aino and Fuegian crania for further comparison.

Let us now consider the horizontal section. This as we have stated is taken through the glabella, while the cranium is arranged so that the plane of the drawing is parallel to the standard horizontal plane. Now the glabella is not a well-defined point, but rather a small area, and a little personal equation will make a considerable change in appearance of the horizontal section according as the plane of section recedes very little from the zygomatic and approaches slightly to the stephanic region. But as the horizontal sections discussed in the present paper are all drawn by members of the same school we are fairly certain that the concavity in the contour behind the temporal line is far more marked in the Moriori than in other crania we have drawn. But it has seemed to us—in view of the importance of the temporal lines from the evolutionary standpoint—desirable in future to take additional coordinates for the points, T_R and T_L†, where the section cuts the temporal line on the frontal bone. Otherwise these important points are lost in the general averaging out of the contours. Allowing, however, for some difference of treatment our contours as well as our plates (see especially Plates II, III, VII, IX, etc.) indicate how marked is the *linea temporalis* of the Moriori skull, and we think that this is an essential factor of our appreciation of its primitive character. If the reader will compare our Moriori horizontal contour with those of the Eskimo, Bantu and English crania, he will realise how relatively great is the depression of the *facies temporalis* in the Moriori frontal bone. A similar opinion will be formed by comparing the photographs of English crania on Plates II and V of Macdonell's memoir‡ with the corresponding photographs of Moriori crania on Plates IX, XV, XVI and XXI, where the shadows indicate the massive character of the *linea temporalis*. This depression of the *facies temporalis* so noteworthy in the Moriori cranium is recognisable in the Cro-Magnon contour, but scarcely more than indicated in Egyptian, Congo and English crania. Now if the reader will superpose the English type horizontal section§ on the Moriori so that the point *V* falls on *F* and *O* on *O*, he will perceive that it

* The resemblance of 17th century English to the Guanche type has already been insisted on: see *Biometrika*, Vol. VIII. p. 135.

† The general asymmetry of the Moriori cranium is well illustrated by the fact that T_R lies posteriorly to T_L in the type cranium: anteriorly the ordinates show that the left side, posteriorly the right side of the Moriori skull is larger.

‡ *Biometrika*, Vol. III. p. 191 *et seq.*

§ Tracing Fig. XXIX of Crewdson Benington's paper, to which the other figures when attached to tracings refer.

is at ordinates 3 and 4 that the maximum difference takes place; it is the depression of the *facies temporalis* which is the marked differential feature of the two contours. If the Guanche contour (Fig. XXVI) be superposed in the same manner, the depression is substantial, but not so marked. When we turn to the Egyptian and Congo contours (Figs. XX and XI), we must adopt a somewhat different superposition. We place *O* on *O* and *OF* in the direction *OF*. We find at once that the posterior portions of these contours fit surprisingly close to the Moriori contour, but that the massive frontal development of the Moriori is completely lacking. To bring out this special feature of the Moriori we may form a *Temporal Index* by measuring the percentage that Ordinate 3 is of the length of the section. We have the following results:

Race	Temporal Index	$100 . \frac{\text{Ordinate 3}}{\text{Minimum Frontal Breadth}}$
Moriori	50·9	99·1
Cro-Magnon	55·7	—
Egyptian	56·2	112·0
Eskimo	57·2	—
Congo, Bantu*	57·5	104·8
Guanche	58·3	—
English	58·8	111·6

There are clearly somewhat mixed factors at work here, but the essentially differentiated character of the Moriori cranium is well-marked. It is the development of the fronto-temporal regions which especially distinguishes the crania of the English commonalty and the Guanche crania from the Moriori.

We now turn to the sagittal contours, where, as we have already remarked, we have not unfortunately the basal triangle and occipital portion of the Benington contours for full comparison. The first problem is: Which standard line is to be superposed? We have actually the inion, the lambda, the bregma and the nasion as definite points, and the glabella and occipital as less definite localities. Schwalbe takes the glabella-inion line, Klaatsch the glabella-lambda line; we may also take the nasio-lambda line, or the nasio-gamma line, which is a line through the nasion parallel to the Frankfurt standard horizontal plane, meeting the occipital in γ. Personally we doubt the value of any line drawn to either glabella or inion, as applied to the *individual* cranium; the terminals are too vague, although doubtless when we deal with a type contour, these so-called points average out to fairly reliable positions. The glabella-lambda line has been asserted to be nearly parallel to the standard horizontal plane†, but we know of no properly drawn type contour in which the height of the lambda is not two to three times as great as the height of the glabella above the nasio-gamma line. In default of

* Fernand Vaz, 1864 Series.

† Quain's *Elements of Anatomy*, Vol. IV, Part I. p. 132.

other points on defective portions of crania either glabella or inion may be used. About the standard horizontal plane three statements may be made in the case of type crania:

(i) It passes very nearly through, but a little below the inion.

(ii) It is approximately parallel to the basio-opisthion line, i.e. parallel to the plane of the *foramen magnum*.

(iii) It is approximately parallel to the maximum length, i.e. the glabella-occipital line.

None of these statements are of course true of the individual skull, but on an average or in type cranial contours they are nearly true as the reader will observe if he examines the Moriori type contours.

If we put aside the glabella as an indefinite "point," and the inion as not available in our comparative contours, we are left with three available lines for superposition: (i) $N\beta$, the nasio-bregmatic line; (ii) $N\lambda$, the nasio-lambda line; and (iii) $N\gamma$, the nasio-gamma or standard horizontal. We will examine the comparison by each of these lines as superposition lines. Let us first, however, recall to mind the essential features of the Moriori skull as revealed by measurement, simple inspection and photograph. These are the flattening of the frontal region and the sagittal ridge or crest behind the apex. We superpose first the English typical sagittal contour (Fig. XXVII). Nasion on nasion and the lines $N\gamma$ in the same direction. We have at once brought out the Moriori characters, the flattening of the frontal and the sagittal crest starting at the bregma or before, eaching behind the apex and extending towards the lambda. There is no question of these striking features. Rotate the tracing until $N\lambda$ coincides in direction with $N\lambda$, all trace of the crest disappears, there is increased frontal flattening, but it is the English skull which now might be thought crested! Now make $N\beta$ coincide with $N\beta$. Here we get a really true measure of the flattening of the frontal bone, but practically the English and Moriori contours now coincide from bregma to lambda. In our opinion there is not the least doubt that comparison by the nasio-gamma line, not the nasio-lambda line, brings out the characteristic features of the Moriori cranium in a manner which fails entirely when we use the nasio-lambda line. Even in the question of the appreciation of frontal flattening, it is by no means certain that the flattening of the frontal bone is the sole source of the physiognomic impression; the rotation of the frontal bone as a whole (which would not alter the ratio of subtense to nasio-bregmatic chord (see p. 110)) may have equal physiognomic value. In the present case this physiognomic factor gives an aspect of flatness to the English skull, although there is a high value of the nasio-bregmatic frontal index. Anyone looking at the photographs in Macdonell's memoirs on the English skull* would assert that there is physiognomically a flattened frontal. It is largely produced by rotation of the frontal bone and not by its flattening.

* See *Biometrika*, Vol. v. p. 104, Plates XII, XVI, etc.

All these points are well brought out when we compare by the standard horizontal planes and have no suggestiveness at all when we superpose the nasio-lambda lines, which give the English skull a protrusion of the frontal *both* by rotation of $N\beta$ and curving of the frontal bone, while actually the two factors act in opposed directions.

Another point which tells considerably against the use of the nasio-lambda line as reference line in the case of two type contours is the experience that this superposition usually throws out very much the two basal triangles (i.e. nasion, basion, alveolar point). As a matter of fact when the basal triangles are as nearly as possible superposed, then the nasio-bregma lines will be closest together, next the nasio-gamma lines and last the nasio-lambda lines, but sufficient material is not yet available to study properly this point, and accordingly no stress is laid on it here.

If the Guanche tracing (Fig. XXIV) be superposed on the Moriori, N on N and $N\gamma$ along $N\gamma$, we have the same result, the frontal flattening and the post-bregmatic crest; this is equally so if $N\beta$ be set to $N\beta$, but the crest disappears entirely, if we set $N\lambda$ to $N\lambda$. When we compare the Negro contour (Congo Bantu, Fig. IX) with the Moriori we have still, for $N\gamma$ to $N\gamma$, the flattening of the frontal ($N\beta$ almost coincides with $N\beta$) and the emphasis of the crest, but a new feature comes in, namely the dwarfing of the occipital portion of the Negro cranium. To a less marked extent this occurs with the Egyptian type contour (Fig. XVIII), but the emphatic flattening of the Moriori frontal bones appears reduced in this case by the physiognomic flattening of the Egyptian due to rotation of the frontal bone.

From other aspects there are considerable objections to the use of $N\lambda$ as a reference line especially in the case of *individual* crania. While the suborbital and auricular points are capable of fairly accurate determination when the skull is on the craniophor and the bregma likewise as the coronal suture is not usually deeply serrated, the lambdoid suture is often most markedly serrated and the actual position of the lambda may be the result of a random serration, which appears to have little bearing on the average sweep of the lambdoid suture, and the intersection of that sweep with the sagittal suture.

It is not possible to compare the constants of the type sagittal section of the Moriori as found for Schwalbe's glabella-inion line with the like constants for the races dealt with in Benington's paper, for the position of the inion has not been averaged off from his individual contours. We shall accordingly content ourselves with comparison of the indices obtained from bregmatic subtense and maximum subtense on the nasio-gamma, the nasio-lambda and Klaatsch's glabella-inion lines as base, giving further the angles which the nasio-bregmatic or glabella-bregmatic line as the case may be makes with these base lines. We shall still have to confine our attention to male crania as the Benington paper has not provided the contours for female crania.

Table V results:

TABLE V.

Values of Constants of Sagittal Contours.

Race	Base Line	Max. Subtense		β Subtense		Index Max. Sub.		Index β Sub.		Δ (Ind. Max. Sub. – Ind. β Sub.)		Bregmatic Angle
	B_o	y	x	y	z	y/B_o	x/B_o	y/B_o	z/B_o	y/B_o	x/B_o	
	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)	(xi)	(xii)
A. *Nasio-gamma Base Line.*												
Eskimo	186·1	93·9	104·1	89·3	72·1	50·5	55·9	48·0	38·7	2·5	17·2	51°·1
Moriori	180·2	90·0	103·8	84·9	73·0	49·9	57·6	47·1	40·5	2·8	17·1	49°·3
English	183·5	84·9	103·7	83·0	77·2	46·3	56·5	45·2	42·1	1·0	14·4	47°·1
Guanche ...	183·4	84·8	99·0	83·0	74·4	46·2	54·0	45·3	40·6	0·9	13·4	48°·1
Egyptian* ...	181·9	84·4	106·2	82·5	79·0	46·4	59·5	45·4	43·4	1·0	16·1	46°·2
Congo, Bantu† ...	175·6	85·4	97·7	83·5	70·6	48·6	55·6	47·6	40·2	1·0	15·4	49°·8
B. *Nasio-lambda Base Line.*												
Eskimo	183·1	71·2	89·5	71·2	88·5	38·9	48·9	38·9	48·3	0	0·6	38°·8
Moriori	180·0	66·0	100·2	66·0	91·0	36·7	55·7	36·7	50·6	0	5·1	35°·9
English	181·2	68·7	90·0	68·7	90·0	37·9	49·7	37·9	49·7	0	0	37°·4
Guanche... ...	181·5	68·5	79·8	68·1	88·0	37·7	44·0	37·5	48·5	0·2	–4·5	37°·7
Egyptian* ...	180·1	70·2	85·2	70·0	89·6	39·0	46·8	38·9	49·8	0·1	–3·0	38°·0
Congo, Bantu† ...	176·3	69·8	90·1	69·8	84·0	39·6	51·1	39·6	47·6	0	3·5	39°·7
C. *Glabella-lambda Base Line.*												
Eskimo	184·5	66·0	100·4	65·2	87·1	35·8	54·4	35·3	47·2	0·5	7·2	36°·8
Moriori	181·0	61·2	102·0	60·6	88·7	33·8	56·4	33·5	49·0	0·3	7·4	34°·3
English	182·8	65·7	92·5	65·6	88·6	35·9	50·6	35·9	48·5	0	2·1	36°·5
Guanche... ...	183·2	64·6	82·3	64·6	82·3	35·3	44·9	35·3	44·9	0	0	38°·1
Egyptian* ...	180·9	66·8	91·5	66·8	88·9	36·9	50·6	36·9	49·1	0	1·5	36°·9
Congo, Bantu† ...	176·6	64·7	91·7	64·3	80·9	36·6	51·9	36·4	45·8	0·2	6·1	38°·5

* XXVI—XXX Dynasties. † Fernand Vaz, 1864 Series.

In the above table columns (i)—(v) give absolute measurements, (vi)—(xi) provide indices and (xii) the "bregmatic angle," or angle the base line makes with the line joining nasion or glabella to bregma. It has been suggested that this angle measures in some manner the flattening of the frontal. It seems to us incapable of really determining the grade of primitiveness in the cranium. Other things being equal this angle will merely measure the extent to which the bregma is thrown back on the cranial vault, a relatively large frontal bone will in itself produce a small bregmatic angle. This is seen at once by comparing the relative position of the bregma as measured by the index x/B_o for the bregma (column (ix)):

Nasio-gamma Base Line.

Race	Angle	x/B_0	y/B_0	Position as to size
Egyptian	46°·2	43·4	45·4	Fourth
English	47°·1	42·1	45·2	Second
Guanche	48°·1	40·6	45·3	Third
Moriori	49°·3	40·5	47·1	Fifth
Bantu	49°·8	40·2	47·6	Sixth
Eskimo	51°·0	38·7	48·0	First

We see at once from this table that the bregmatic angle has little if any relation to size of the skull. It gives absolutely the same order as the relative horizontal position of the bregma, and (except for the anomalous Egyptian, which is, however, greater than the English and Guanche by a slender difference) the same order as the bregmatic height difference. In biometric language we should say that the bregmatic angle is highly and negatively correlated with the horizontal position of the bregma, and is highly and positively correlated with the vertical position of the bregma. These results might be anticipated from its functional form $\tan^{-1}(y/x)$, but we shall learn little as to racial order from the bregmatic angle. Indeed we might even state the more primitive the race the bigger the bregmatic angle. This conclusion is amply confirmed by the Cro-Magnon skull which gives:

Angle	x/B_0	y/B_0
55°·2	34·6	49·6

Thus we must conclude that a high bregmatic angle is the result of a relatively forward bregma and is a sign of primitive rather than advanced cranial type. The reader must carefully distinguish between a "forward" bregma and a relatively large frontal bone. If we take as measure of the latter, the base $N\beta$ of the frontal bone in proportion to $N\gamma$, the order is:

Congo, Bantu ...	63·1	English ...	61·5
Egyptian ...	62·5	Cro-Magnon ...	61·0
Moriori ...	62·1	Guanche ...	60·5
Eskimo... ...	61·5		

an order, to which it would be difficult to give any interpretation of value.

When we turn to either the nasio-lambda or the glabella-lambda as base line we find none of the above identical order of vertical index, horizontal index and bregmatic angle. We do now find that the Moriori have a low instead of a high bregmatic angle, but if we were to associate this with a racially primitive condition, then the Congo Bantus would stand as the most advanced race in our list judged from either base line. As a matter of fact we see the explanation lies in the fact that the Moriori have from the nasio-lambda base the relatively least forward, and the Bantus the most forward position of the bregma, while the same is nearly

true for the glabella-lambda base. The results found for these two base lines whether we consider the bregmatic angle or the horizontal or vertical bregmatic index are not in any accordance, and although these base lines must be used when the subject consists merely of a skull cap, or even lacks the nasion, we doubt whether for racial inter-relations they are nearly as valuable as the nasio-gamma base. The general measure of the flattening of the frontal bone is best read off as on our p. 95, and this gives the Moriori the premier position on the list. If we desire to measure the total physiognomic flatness we might, I think, combine the bregmatic angle for the nasio-gamma line with the angle the line joining the nasion to the arc end of the nasio-bregmatic subtense makes with the nasio-bregmatic line. The following table brings out the contributions to frontal flattening due (i) to flattening of the frontal bone and (ii) rotation of the frontal bone as a whole.

Race	Frontal Bone Flatness	Rotation of base of Frontal Bone	Physiognomic Angle of Flatness
Cro-Magnon	24°·7	55°·2	79°·9
Eskimo	28°·0	51°·1	79°·3
Bantu	27°·3	49°·8	77°·1
Guanche	26°·4	48°·1	74°·5
English	26°·6	47°·1	73°·7
Moriori	22°·3	49°·3	71°·6
Egyptian	24°·9	46°·2	71°·1

This table seems of some interest. As we have before noted the physiognomic flatness consists of two factors; of these the rotation is roughly twice as important as the actual flattening of the frontal bone. Thus the English low physiognomic flatness is largely due to rotation; the Cro-Magnon skull with much frontal flattening is physiognomically at the top of the list owing to the excess of rotation, and in appearance has consequently a well-developed frontal. The Moriori stands at the bottom of the list for frontal bone flattening, but has less physiognomic flatness than the Egyptian, although the difference is very minute. It is clearly impossible to judge any grade of "primitiveness" by the mere smallness of the bregmatic angle, and we venture to hold that physiognomic flatness must in future be distinguished from flatness of frontal bone.

Returning to our Table V, we note that in the case of the nasio-lambda base the maximum subtense for all practical purposes is the bregmatic subtense. In other words if the nasion, but not the lambda, be ascertainable on a skull cap a *rough* approximation can be made to the nasio-lambda line by drawing through the nasion a parallel to the tangent to the sagittal arc at the bregma. The reader will see by examining Δ Index x/B, column (xi), Table V, that the maximum falls sometimes on one side, and sometimes on the other of the bregma, but its position never more than half-a-dozen millimetres from the bregma, causes the two subtenses to be equal owing to the general flatness in the region of the bregma. The maximum

difference between the two subtenses is 0·2 for the Guanche type contour, a difference well within our probable error of determination. If we use the glabella-lambda base line very little difference can be made between the maximum and bregmatic subtenses, although the maximum subtense now always falls behind the bregma, but by something less than 8 mm. As a matter of fact if we set off on the contour βf anteriorly and βl posteriorly to bregma both equal to about 15 mm., then the parallel to fl will give *approximately* $N\lambda$ and the parallel to βl the glabella-lambda line. While these constructions are merely approximate for type contours, they will be only very rough for individual crania, yet in default of better rules they will give some idea of the position of the λ, if only the bregma and either the nasion or glabella are known. If we want the actual position of the lambda we can only note that the foot of the bregmatic subtense is at an average distance ·491 of the nasio-lambda line from the nasion, and at an average distance ·474 from the glabella in the glabella-lambda line, but these distances vary much for individual races, and *a fortiori* will vary more for individual crania*.

Now the point just discussed, i.e. the practical equality of the bregmatic and maximum subtenses for both the nasio-lambda and glabella-lambda cases, suggests how important it is to use a base line which will determine the extent to which the vertex of the cranium rises above and recedes behind the bregma. For the rising of the parietals above the frontal bone, throwing the vertex far back, is one of the marked features of primitive man. Whatever may be said against the Frankfurt standard horizontal plane—and many of the criticisms arise from insufficient experience of its value—it can hardly be disputed that on the *average*, i.e. in type contours, it is a fair approximation to the "horizontal," remembering that even for the same individual the "horizontal" is only itself a sort of average position. Some plane must be taken to express the marked physiognomic characters conveyed by the idea of a retreating vertex and an elevated vertex. This no nasio-lambda or glabella-lambda line can accomplish. As far as we can find no very definite statement as to how the vertex is to be determined has yet been accepted†. We define the "apex" to be the point of the sagittal suture vertically above the auricular line when the skull is adjusted to the standard horizontal position on the craniophor. The "vertex" is the highest point of the sagittal or medial plane when the skull is in the same position. It is the point of contact of the tangent to the sagittal contour drawn parallel to the nasio-gamma line. From this standpoint we are able to give quantitative measures to the elevation and recession of the vertex relative to the bregma. All we need do is to subtract the indices y/B

* Thus we attempted to reconstruct the Cro-Magnon skull from a knowledge of glabella and bregma. A very poor value of λ was found from ·474, but an excellent one when the skull was assumed to be Guanche. In any such reconstructions it is probably desirable to use any suggestion of racial affinity.

† See Quain's *Anatomy*, Vol. IV. Part I. p. 124 (1915), for example, where the vertex is defined to be the "highest point of the arch of the sagittal section." In the diagram, Fig. 104 of this section, the nasio-lambda line appears to slope downwards instead of upwards and the "vertex" for this position appears to be in front of the bregma. In Fig. 112 the nasio-lambda line is made nearly horizontal and this adds much to the prognathous physiognomic effect aimed at!

and x/B of the vertical and bregmatic subtenses. There results the following table:

Race	Elevation	Recession
Moriori	2·8	17·1
Eskimo	2·5	17·2
Egyptian, XXVI–XXX Dyn.	1·0	16·1
Congo, Bantu	1·0	15·4
English	1·0	14·4
Guanche	0·9	13·4

The Cro-Magnon cranium stands between the Eskimo and Egyptian with a recession of 17·2 and an elevation of 1·5. Owing to the flatness of the skull in the neighbourhood of the vertex, the values of the recession can only be looked upon as approximations. Far more racial determinations are desirable before conclusions can be drawn, but undoubtedly these measures of the elevation and recession of the vertex suffice to differentiate those races like Moriori and Eskimo in which a receding sagittal crest is a marked feature.

As a last feature of the sagittal section we may consider the angle between the nasio-lambda line and the standard horizontal. We have found no case of the depression of this line below the nasio-gamma line; the racial elevation is always about 9° to 13°. We have:

Race	Slope of Nasio-lambda Line
Moriori	13°·5
Cro-Magnon	12°·6
Eskimo	12°·3
English	10°·5
Guanche	10°·3
Congo, Ferdinand Vaz, 1864	9°·9
Congo, Batetelu	9°·9
Egyptian, XXVI–XXX Dyn.	8°·4

Thus we again see the Moriori and Eskimo at the top. It might be thought accordingly that the elevation and recession of the vertex in Moriori and Eskimo was due to a mere tilt of the cranium. The reply to this is that there is an actual physical ridge, there being actual concavity in some cases of the parietals on either side of the sagittal suture.

A useful piece of work would be to add the basion, the inion, the opisthion and the suborbital and auricular points to Crewdson Benington's contours—they are given on his individual tracings; so that racial comparisons might be made of the lower portion of the cranium from his sagittal type contours.

7. *On Mean Values found from Type Contours.* We now turn to the second problem relating to the type contours: How far can we be certain of mean values obtained from them? We may note several general points before we proceed to

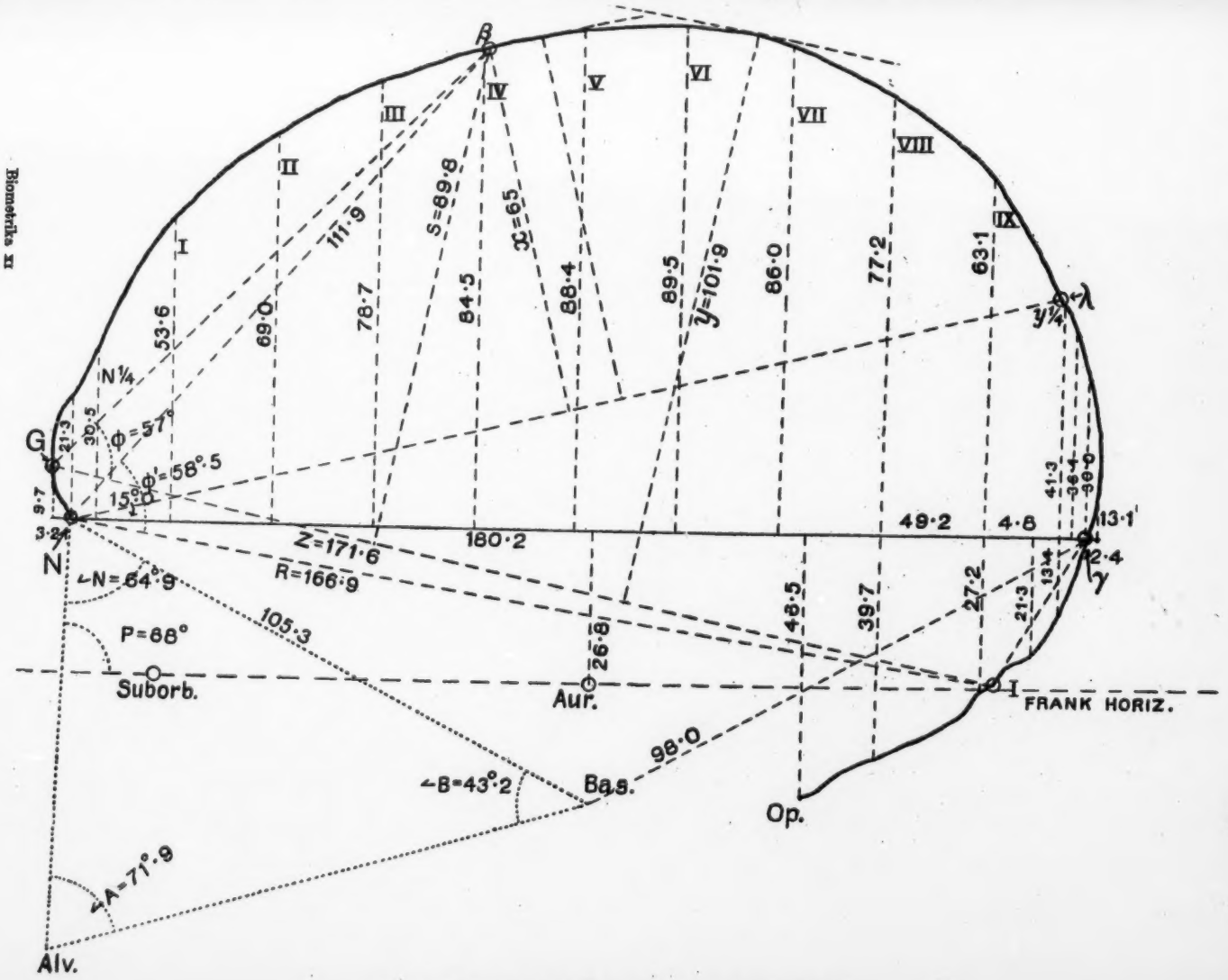


FIG. I. Moriori Type Sagittal Contour. ♂. (From 33 skulls.)

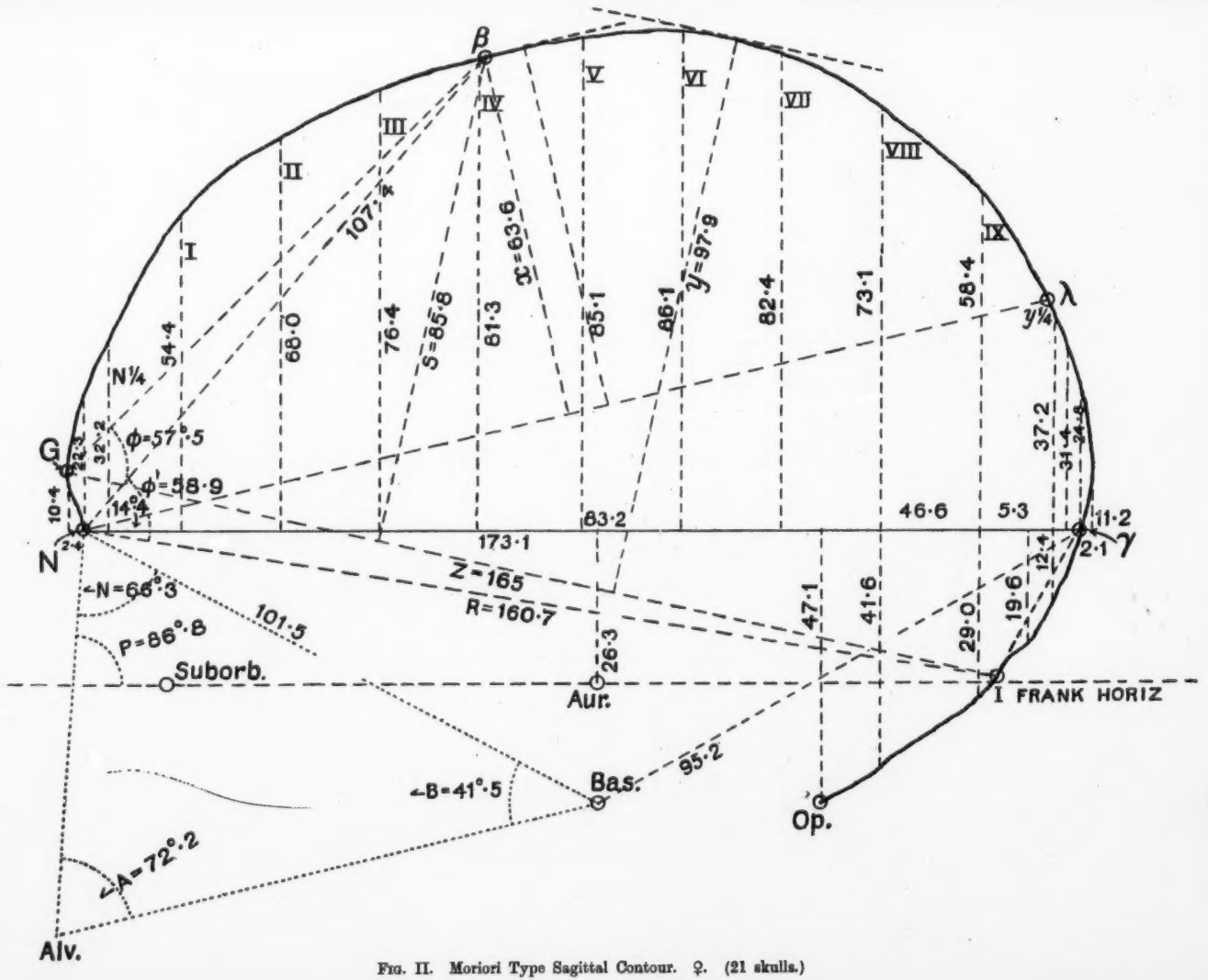


Fig. II. Moriori Type Sagittal Contour. ♀. (21 skulls.)

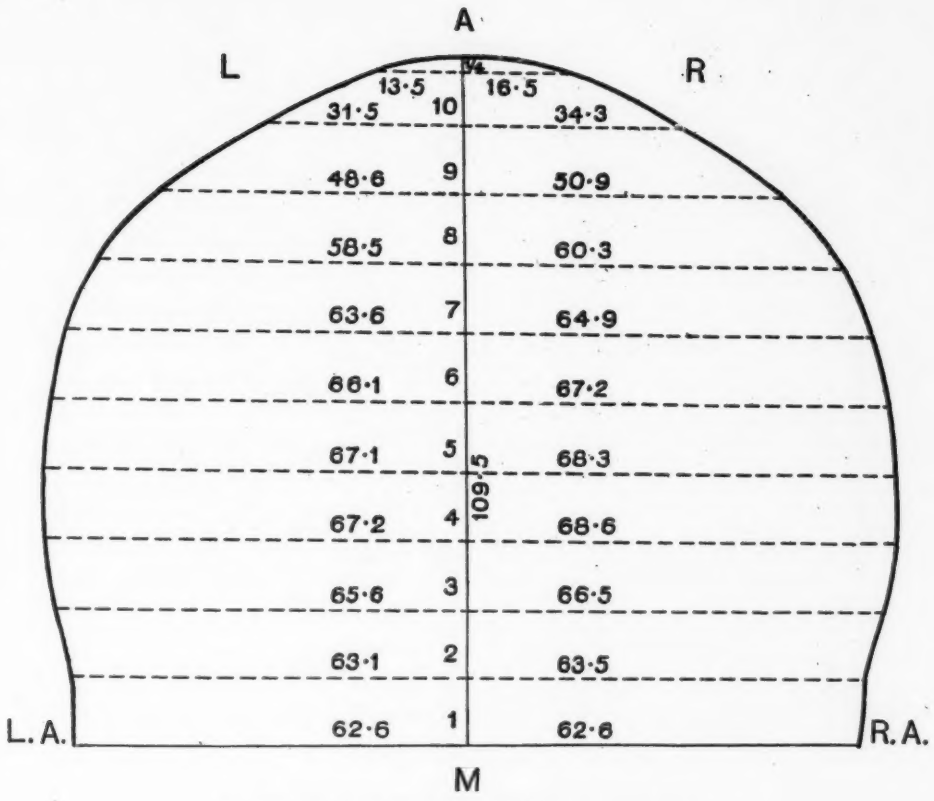


Fig. III. Moriori Type Vertical Contour. ♂. (33 skulls.)

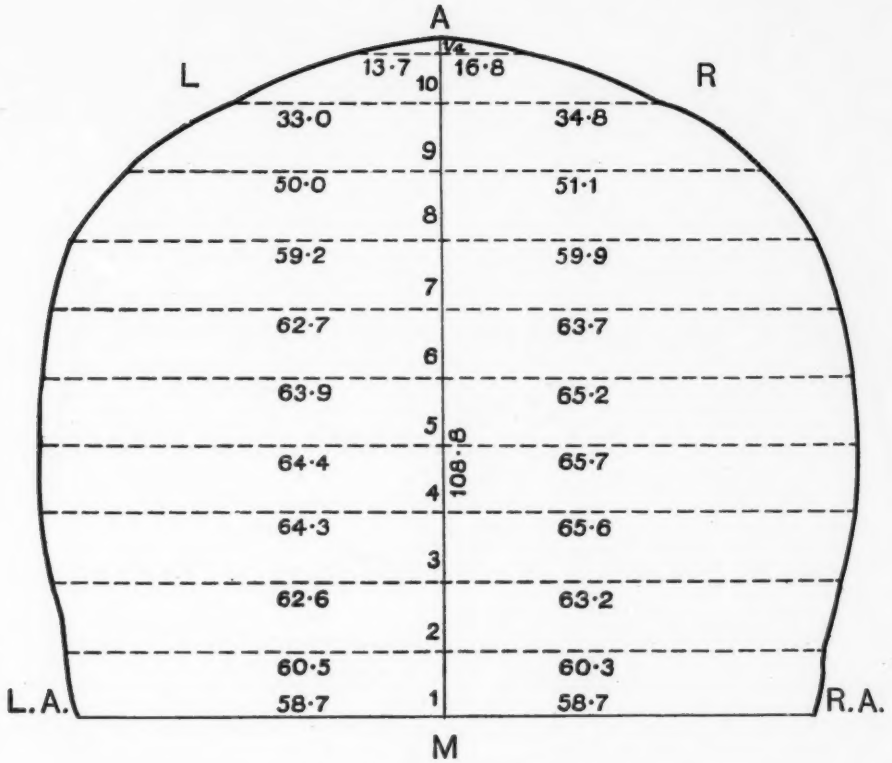


FIG. IV. Moriori Type Vertical Contour. ♀. (21 skulls.)

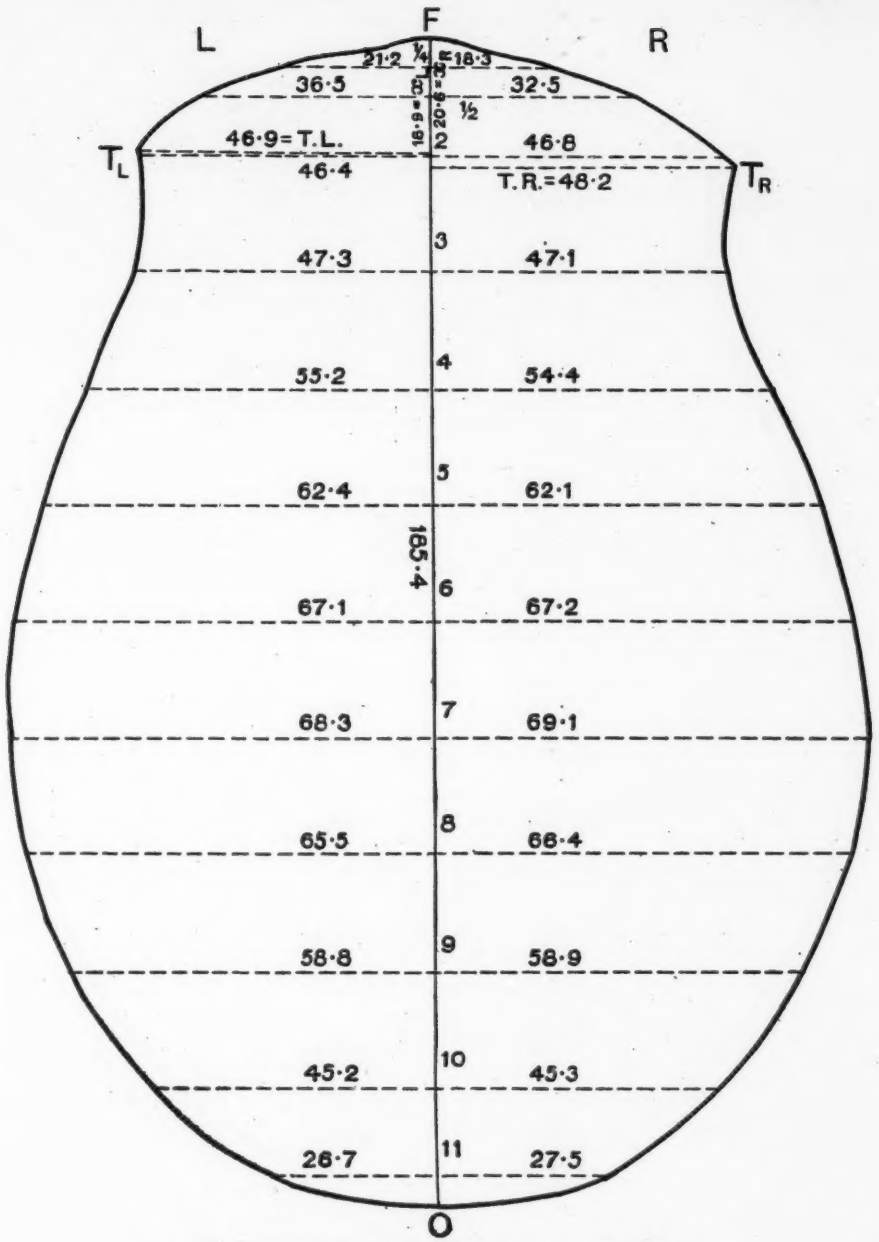


FIG. V. Moriori Type Horizontal Contour. ♂. (33 skulls.)

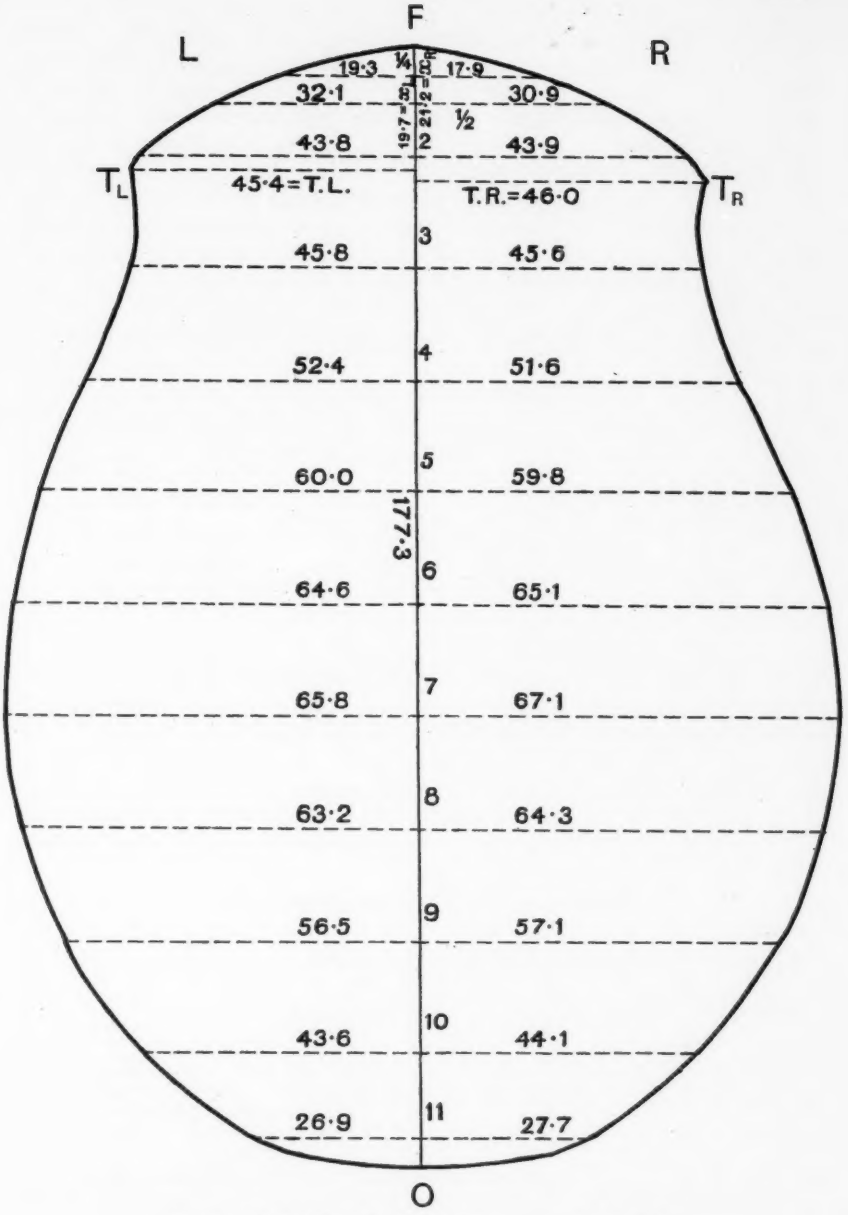


FIG. VI. Moriori Type Horizontal Contour. ♀. (21 skulls.)

details. After all the values have been averaged, some error is almost invariably introduced in plotting, especially as the contours have to be somewhat thickened up for the engraver; thus readings should be made to the *middle* of the contour boundary. But even then with the greatest care the engraver seems to find it difficult to reproduce to exactly the same size; however strict the injunctions may be we rarely find the engraver's final result to agree absolutely with the plotted contour. There may indeed be as much as a millimetre difference. In deducing new indices therefore it is well to read off the diagrams, disregarding the numbers placed upon them and take the ratio of lengths thus read. We will now turn to the type contours. Let us consider first the glabella-occipital length. We ought to be able to get this exactly from the sagittal mean sections, and approximately from the horizontal sections. There results by measurement:

Mean glabella-occipital Length L.

	Males		Females	
	No.	Value	No.	Value
Direct Measurement ...	35	186·9	22	178·6
From Sagittal Contour ...	33	186·5	21	178·3
From Horizontal Contour	33	185·4	21	177·3

That the horizontal contour does not give the same result as the sagittal is to be anticipated, for the horizontal line through the glabella does not necessarily pass through the exact occipital point, but the measurement on the sagittal section should theoretically agree with direct measurement. Is the slight difference due in this case to the fact that one or two skulls could not be used for type contours? Unfortunately the type crania in the earlier cases were not finished so fully in the occipital region as the present samples, but I give the following examples:

Mean glabella-occipital Length L for Males.

Race...	English		Egyptian, XXVI –XXX Dyn.		Congo, Batetelu		Gaboon, Bantu	
	No.	Value	No.	Value	No.	Value	No.	Value
Direct Measurement ...	137	186·9	100*	184·1	50	177·8	50	179·5
From Sagittal Contour ...	100	186·8	100	184·4	41	178·4	50	177·9
From Horizontal Contour	100	185·9	100	183·5	41	177·5	50	177·6

* The 100 ♂ crania to which direct measurement was applied were not absolutely the same as the 100 selected for contouring. Fawcett gives 185·1, MacIver 184·5 for prehistoric Egyptians.

I think we may conclude from these results that:

(i) It is needful to add about 1 mm. to make the measurement on the horizontal contour agree with that on the sagittal contour.

(ii) Unless the occipital and glabella are reconstructed with great accuracy, we may have an average error of something like 0·6 mm. in determining the glabella-occipital length from the sagittal section.

Of course the maximum breadth *B* cannot be read off on the horizontal section; the temporal breadth should be something *greater* than the minimum frontal breadth; it is for ♂s 95·5 and for ♀s 92·3 against minimum frontal breadth, ♂s 95·3 and ♀s 94·0. I am unable to account for the defect of 1·7 in the females except by the difference of errors of reading with callipers and reading with a scale on the diagrams*. Probably it would be best in future contour drawings with the Klaatsch to mark off specially the temporal ridges, as one is apt in turning a sharp point on the contour to bring the side of the needle rather than its point into contact with the ridge, but I do not think this would *lessen* the dimensions; such a lessening might arise from shear of the plasticine on which the skull is bedded, if too great pressure were applied. Whatever the source of the error, the lesson must be to test each individual contour against the available individual measurement before proceeding to the type contour.

I now pass to the transverse section. In drawing this we pass from "auricular point" to "auricular point," the "auricular line" being defined as the tangent to the upper borders of the external meatus. We have been forced to the conclusion that when the skull rests on the Ranke craniophor, the "auricular line" corresponding to the top of the plugs (wedge type, not conical type) does not coincide with the auricular line determined by joining the upper borders of the external meati as marked on the transverse contour. The ear wedges have to be thrust some distance into the auditory meatus† and thus their horizontal tops are considerably below the line joining the upper borders. They are of course not as low as the line joining the "centres" of the external meatus. Thus it comes about that we can get three auricular heights. The one from the centre of the auricular passages is that usually determined on the living, and by field workers on craniological material; the one given by the contour tracer is that which we ought to find in strict accordance with the Frankfurt Concordat, and the one actually found in laboratory work with a craniophor is only strictly in accordance with the Concordat, when the external meatus has an upper surface, actually horizontal and accurately passing through the uppermost point of the border of the auditory external meatus.

We now turn to the type transverse contours and note in the light of the above remarks the auricular heights:

* The cranium which it was not possible to contour had a large frontal breadth, but its omission only reduces the mean to 93·8 mm.

† In order that the skull may be stable on the craniophor.

Moriori Auricular Height OH.

	Male		Female	
	No.	Value	No.	Value
Measured on Craniophor	34	117·5	22	115·1
Measured on Transverse Contour ...	33	109·5	21	108·8
Measured on Sagittal Contour ...	33	116·5	21	112·4

Let us examine similar data for other races:

Male Auricular Height OH.

Race...	English		Egyptian, XXVI –XXX Dyn.		Congo, Batetelu		Gaboon, Bantu	
	No.	Value	No.	Value	No.	Value	No.	Value
Measured on Craniophor...	135	114·6	100	113·6	47	113·9	?	?
Measured on Transverse Contour	100	111·5	100	110·2	41	112·0	50	113·6

It will be seen that there is not much comparative material, but in all cases the craniophor auricular height is in excess of the transverse contour height. Dr Benington was more successful than I have been in making his type transverse contour approach the auricular height of the craniophor measurement*, but even he may be 2 to 5 mm. in defect. My determinations from the sagittal contour are in better agreement with my craniophor measurements, but may be 2 to 3 mm. divergent, while the transverse contour differs by 6 to 8 mm. I attribute these big differences to the very different manner in which the auricular point is determined (i) when the skull is on the craniophor, (ii) when it is orientated by auricular points and apex, (iii) when it is orientated by nasion, bregma and lambda. In the latter case in particular there is no certainty that the auricular line is perpendicular to the plane of the paper. In (ii) the auricular point is reached by a stile arriving from the apex, in (iii) by a stile adjusted from the basal side of the orientated skull, and there is far more personal equation in its determination than may be at once realised. It is clear, I think, that determinations of the auricular height from the transverse type contour are of small value. But I doubt very much whether the auricular height as found on the Ranke craniophor accords well with the definition of the Frankfurt concordat, or indeed whether

* It is possible that the top of the auricular passage is less nearly horizontal in the case of the Moriori than in the races which Dr Benington dealt with.

the "auricular line" is capable of sufficiently close definition*. At any rate the transverse contours need clearer determination at their lower terminals. In further work it might be valuable to place upon them the projections of the basion, suborbital and alveolar points.

I have not taken directly the interauricular breadth of my Moriori skulls. Read off from the transverse contours, I find: ♂s, 125·2, and ♀s, 117·6, which agree fairly well with Scott's measurements, namely ♂s, 126·8, and ♀s, 118·5, although mine for both sexes are consistently less.

I made an attempt to ascertain the circumferences from the contours and found:

Circumference	Male		Female	
	By Tape	From Contour	By Tape	From Contour
Transverse: Q	316·1	300·1 (316·1)	310·0	297·3 (309·6)
Horizontal: U	522·8	522·0	502·6	501·0
Sagittal: S_1	126·0	125·7	121·4	121·6
Sagittal: S_2	122·4	122·9	117·4	117·4
Sagittal: S_3	121·0	118·9	116·0	110·0
Sagittal: $S = S_1 + S_2 + S_3$	369·4	367·5	354·8	349·0

After the contour measurements for Q are placed in brackets the results which would be reached by adding to the contour arc *twice* the difference in the auricular heights. The agreement is then very close, showing that the divergence is solely due to difference in position of the auricular line and not to shape of contour. The results on the whole are fairly close; the occipital arc is the worst, and I have possibly not accentuated sufficiently the ridges of the occipital and inionic regions†.

I now give (Table VI) a number of other characters which can be read off the type sagittal contour with comparison of their measured values. On the whole the agreement is not bad. Indeed it is sufficiently good to suggest that with the experience now gained an improved technique would enable the sagittal type section to be directly used as a means of determining the customary racial characters of the skull, or that at all events this section would serve as a valuable control method for verifying the mean values determined by direct measurement and arithmetic. The measurements of greatest divergence are:

(i) The foraminal length. The type contour exaggerates it by 2 mm. about for men and 4 mm. for women. But the draughtsman has plotted in the case of the female skull the opisthion with an abscissa of 44·6 instead of 46·6 mm. from the gamma. Clearly the error observed in the occipital arcs is largely due to this displacement of the opisthion 2 mm. in the male and 4 mm. (2 of which are by

* It is proposed to follow this matter up by actually continuing the tracing of the transverse section into the auditory meatus to the limit of the Klaatsch tracer's possibilities and comparing with the craniophor auricular heights.

† See, however, the remarks later on this point.

TABLE VI. *Measurements from Sagittal Type Contours.*

Character	Male		Female	
	By Cranial Average	By Type Contour	By Cranial Average	By Type Contour
F	182·0	182·2	176·1	175·8
I'	185·6	185·6	177·8	178·2
H	135·9	135·4	132·8	131·4
LB	105·6	106·0	102·3	101·5
G'H	76·4	77·0	70·0	71·1
GB	103·2	101·0	96·6	97·5
fml	35·3	37·2	34·4	38·5
Oc I	62·2	62·1	64·2	64·2
Frontal Index	21·6	19·4	21·9	21·1
N∠	64°·9	64°·8	65°·7	66°·3
A∠	71°·8	71°·9	73°·6	72°·2
B∠	43°·4	43°·2	40°·7	41°·5
P∠	84°·7	88°·0	84°·5	86°·8

draughtsman's error) in the female. I take it that in the measurement of the foramen with the dividers the tendency is to measure the inside border of the foramen, but with the Klaatsch contour tracer to stop just short of the edge. It seems to me reasonable to suppose that if the basion and opisthion were first marked off as points and the foraminal length then compared with actual measurement, we should insure not only the exactitude of the foraminal length, but of the occipital arc.

(ii) The profile angle. Here the difference is 3°·3 in the males and 2°·3 in the females; this is serious. I take it to arise from a divergence in the determination of the alveolar point (i) with the sagittal contour vertical and using the goniometer, and (ii) with the sagittal contour horizontal and using the Klaatsch. But I must reserve the discussion of the point until further experiments have been made. The fundamental triangles are in quite good accordance, and accordingly the evil may lie in the determination of the auricular point, when the skull is off the craniophor. We note how closely the occipital indices deduced from the contours approach those obtained from the measurement of occipital arc and chord. Indices from the contours were taken as i = occipital subtense/occipital chord. Then if I be the occipital index as defined by

$$I = 100 \frac{S_3}{S'_3} \sqrt{\frac{S_3}{24\,(S_3 - S'_3)}},$$

it is easy to deduce that

$$I = 100 \left(\frac{1}{2} i + \frac{1}{8i}\right).$$

The occipital indices so far obtained are:

		Male		Female	
English (Moorfields)		58·5		59·2	
Moriori		62·2		64·2	
Gaboon, Bantus	68·3	68·5		69·2	68·1
Congo, Batetelus	68·8			67·0	

The smaller the index the smaller is the radius of curvature of the occipital arc in relation to its chord, i.e. the occipital is more rounded. The order shows that the English have most, the Negro least occipital development, and that the Moriori male stands nearer to the European than to the Negro.

The Frontal Index is on another footing. If the nasio-bregmatic chord and arc be given, i.e. S'_1 and S_1, then

$$I_f = 100 \frac{S_1}{S'_1} \sqrt{\frac{S_1}{24 (S_1 - S'_1)}}$$

gives the frontal index in the form of the ratio of the radius of curvature of the frontal arc to the frontal chord. To obtain this index the distance from nasion to bregma along the chord should be measured with the callipers. Not having measured this, I took the values from the contours which should be extremely close to the measured values as the terminals are very definite points. The results were:

Males, 68·7; Females, 67·9.

If now we wish to replace the index by $i_f = 100 \times$ subtense/chord, we must deduce i_f from

$$i_f = 100 \left(\frac{I_f}{100} - \sqrt{\left(\frac{I_f}{100} \right)^2 - \cdot 25} \right).$$

There resulted the values 21·6 and 21·9 to compare with the direct contour values of i_f, 19·4 and 21·1. The reason for the excess is clear: it is due to the exaggeration of the length of the frontal arc (treated as a circular arc) owing to the massive glabella, more divergent in the case of the male than the female*. On the other hand if we take English males, we find from arc and chord measurements of the frontal $I_f = 70{\cdot}1$ leading to $i_f = 21{\cdot}0$, which is in closer accord with 22·2, the direct value of the subtense to chord index, thus indicating that with less glabellar prominence, the results of the two methods would be more nearly equal and would more closely justify the assumption of the approximate circularity of the frontal arc†. On the whole if the frontal curvature is to be determined by direct measurement, i.e. not by contour drawing, it will be desirable to adopt a modified form of Merejkowsky's simometer‡. It would not be hard to devise, but it would have to be considerably larger, and the subtense would have to be obtained by contact—it would not necessarily be at the bisector of the chord.

* Had we supposed the arc to be catenary rather than circular the values (see *Tables for Biometricians and Statisticians*, p. 64) would have been 22·7 and 23·1, still more divergent. A parabolic arc was found to be even less successful.

† Tasmanian aborigines (Berry and Robertson, *R. S. Edin. Proc.* Vol. XXXI. Table p. 60) give for combined sexes, from I_f (64·91), $i_f = 22{\cdot}33$.

‡ *Biometrika*, Vol. VIII. p. 317.

To sum up: the comparison we have made suffices to show that cranial type contours can be of great service; they are justified by the generally good concordance they give when tested against average measurements of the skull made in the ordinary manner. But if they are to serve for the purpose of accurate measurement and for more than a rough comparison of race with race, we think a considerably improved technique will be essential and if, as we believe, it is desirable to compare the contours of the individual crania step by step with their actual measurements, then the contour drawing will not relieve us from the usual craniometric processes, but at first serve only as a control. In particular the difficulties which have arisen in our own case centre especially round: (*a*) a better agreement in the position of the auricular point as determined (i) by the craniophor and (ii) on the transverse and sagittal contours; (*b*) in the better location of the opisthion and alveolar point on the sagittal contour.

These do not seem substantial difficulties, but they need practice and caution. The basion can always be inserted from the nasio-basionic length and basio-bregmatic lengths (*LB* and *H*); the opisthion from the foraminal length and the lambda opisthionic chord (*fml* and S'_3). The alveolar point is then determined by the upper face height (*GH'*) and the profile length (*GL*). To avoid the contour determination of the auricular point, we might mark the apex (*A*) of the individual skull when on the craniophor as well as the gamma. Then on the type contour by drawing a line through the average apex perpendicular to the average $N\gamma$, and plotting off along this line the mean auricular height (*OH*), we should obtain an auricular point which would agree with the cranial measure ents. The parallel to $N\gamma$ through the auricular point thus determined would give the standard horizontal plane, and the profile angles by contour and measurement should now coincide.

A marked feature of the type Moriori sagittal contours for both male and female is evident on inspection—the auricular point is vertically above the basion. It is not true for individual crania, but it may be true for type crania (i.e. on the average) for other races than the Moriori. If so the vertical through the basion would give the apex and the auricular point could be determined from the mean auricular height, and control would be easier to establish. In general the basi-auricular line as an approximate or average vertical of the head deserves consideration*.

* It is proposed shortly to observe skull by skull the difference between the contour and the direct measurements with a view to improving technique. The English male sagittal type contour when the basion and alveolar points were added as above suggested gave a profile angle of 86°·3, according reasonably with the measured angle 86°·1, but the same method applied to the Congo Batetelu cross-section only gave a profile angle of 81°·2 as against 82°·8 measured directly on the crania. The actual marking of apex and gamma when the skull is on the craniophor and the transfer of these marked points to the contour will undoubtedly aid in ascertaining what is needful to bring contour measurement and direct measurement systems into full unison. At the same time it must be remembered that the characters of a mean or type contour cannot be demonstrated theoretically to coincide absolutely with the means of characters not used in constructing the contour.

8. *On the Moriori Mandible.* A difficulty arises in treating mandibles in general from the fact that they are usually very few in number, often separated from the skulls, and then difficult to sex, and further that as many writers neglect them altogether there are few data for comparative purposes*. The present material contains only eleven mandibles and of these one certainly did not belong to the skull with which it was associated. Four were female and seven were male. Scott has fourteen mandibles, twelve male and two female. In the case of the Maori he has 24 male and seven female mandibles. In the following table I have added Scott's data to mine for the Moriori and compare results with Macdonell's for English, Fawcett's for Naqada Egyptians, Koganei's for the Aino, and a few scattered results for Negro races.

TABLE VII. *Measurements of the Mandible.*

Race...	Moriori			Maori			Aino		
Sex ...	♂	♀	♂+♀	♂	♀	♂+♀	♂	♀	♂+♀
Bicondylar Width... ...	124·1	114·8	122·3	127·2	120·9	125·8	102·0	95·6	99·5
Bigonal Width	106·4	101·6	105·3	104·5	97·3	102·8	—	—	—
Height at Middle Incisors	35·3	30·7	34·2	34·5	29·7	33·4	33·8	32·6	33·4
Mental Distance	51·6	49·1	51·0	50·5	49·1	50·2	—	—	—
Total Numbers (round) ...	19	6	25	24	7	31	60	35	95

Race...	English	Gaboon, Bantus			Northern Negro	Fuegian	Egyptian, Naqada		
Sex ...	♂+♀	♂	♀	♂+♀	♂	♂+♀	♂	♀	♂+♀
Bicondylar Width... ...	113·2	112·6	110·0	111·6	117·0	120·6	110·5	106·4	108·2
Bigonal Width	95·4	94·3	90·1	94·4	97·2	103·4	93·6	87·6	90·4
Height at Middle Incisors	30·8	32·7	30·5	31·8	—	35·4	32·9	31·6	32·2
Mental Distance	43·7	45·4	44·3	44·9	—	—	44·4	43·1	43·7
Total Numbers (round) ...	140	29	20	49	16	5	50	60	110

Now these results, slender as are the numbers, seem of extreme interest. All comparison of the English and Aino mandible with that of the Moriori is now out of place. The Moriori, Maori and Fuegian are the only mandibles with a bicondylar width over 120 mm. and a bigonal width over 100. But the most

* The Laboratory has in hand a series of many hundred Egyptian mandibles on which a much fuller system of measurements has been made. It is hoped to reach more definite criteria for sexing from this material.

marked differentiation of the Moriori and Maori from those of other races is the distance apart of the *foramina mentalia* which is over 50 cms. It is a misfortune that we have not this distance for the Fuegian mandibles, in order that we might ascertain whether it is usual for heavy large mandibles to have a mental distance of these dimensions, or whether it is a special racial character in Moriori and Maori. Judged by the general massiveness of mandible the racial order is:

Moriori and Maori—Fuegian—Northern Negro—English—Gaboon Bantu—Egyptian—Aino.

Taken in order of depth of chin, we have:

Fuegian—Moriori—Maori—Aino—Egyptian—Gaboon Bantu—English.

Lastly, if we class these races by the index 100 × bigonal width/bicondylar width, which to some extent measures the verticality of the ramus, we have the order:

Moriori (86·1)—Fuegian (85·7)—Gaboon Bantu (84·6)—English (84·3)—Egyptian (83·6)—Northern Negro (83·1)—Maori (81·7)—Kaffir (81·0).

These orders are not very illuminating except as grouping the primitive races of the southern hemisphere together by massiveness of mandible, and showing the Moriori with the most vertical, the Kaffir with the most sloping ramus. It is clear that two racial types of skull may approach each other fairly closely, while there can at the same time be a wide difference between their mandibles.

9. *Description of and Remarks on the Photographic Plates of Moriori Crania.* Plates II, III and IV show a typical Moriori male cranium, No. 765[45]. Plate II gives the *norma facialis* and indicates the high nose, the massive glabella, the flattened frontal and receding temporals. Plate III, the *norma lateralis*, left profile, indicates not only the same last three points, but the post-bregmatic depression followed by the sagittal crest, and further the massive mandible with vertical ramus. Plate IV, the *norma occipitalis*, marks the pentagonal form of this aspect as well as the generally massive character of the Moriori skull. The apex of the "pentagon" is the result of the sagittal crest.

Plates V and VI, No. 765[10], give another typical male cranium. Plate V, the *norma lateralis*, right profile, shows well the characteristic frontal region and the marked sagittal crest, also general massiveness in the occipital, inion and mastoid regions. Plate VI, the *norma occipitalis*, brings out, in a still more marked manner than even Plate IV, the sagittal crest and indicates well the flattening of the parietals on either side of the sagittal suture.

Plates VII and VIII are again photographs of a male skull, No. 765[25]. Plate VII, the *norma lateralis*, left profile, is an endeavour to catch the marked temporal line and the recession of the temporal bone behind it, which gives the characteristic aspect to the horizontal contour. Plate VIII gives the *norma basalis* and indicates well the large and rotund character of the *foramen magnum*, although the foraminal index of this cranium does not reach the mean value. The heavy inion ridge is to be noted.

Plate IX, No. 765, and Plate X, No. 765^{32}, give the *norma lateralis*, left and right profiles respectively, of typical female crania. We see the same features as in the male, the massive frontal, marked temporal lines, receding *facies temporalis*, and the elevation and recession of the vertex produced by the sagittal crest. These features are less emphasised than in the male, but are in striking contrast to the female characters of less primitive races.

Plates XI and XII provide the *norma verticalis* and *norma basalis* of a typical female, No. 763. In Plate XI we see the pear-shaped vertical aspect*, and in Plate XII the characteristic palate, and the asymmetrical foramen.

Plates XIII and XIV further illustrate in a male cranium, No. 765^{5}, the racial characteristics. Plate XIII shows in the *norma basalis* a marked asymmetry of the foramen as well as the general massiveness of the base. Plate XIV, the *norma lateralis*, right profile, gives the usual primitive factors of massive frontal sinuses, receding forehead, sagittal crest and vertical ramus; but the vertex in this case while markedly elevated does not recede; as the sagittal contour of this cranium indicates it is practically at the apex.

Plates XV and XVI give *norma lateralis*, left profile, and *norma occipitalis* of a male skull, No. 762. Besides all the usual racial features of which the depression of the *facies temporalis* is especially marked, we see a rarely occurring ossicle of the coronal suture, and a protrusion of the occipital almost amounting to bathrocephaly. Plate XVI indicates the source of this in the conglomeration of ossicles in the lambdoid suture.

Plates XVII and XVIII reproduce another male skull, No. 765^{27}, with massive frontal, receding forehead, depressed *facies temporalis* and sagittal crest. Plate XVII, the *norma lateralis*, left profile, shows an epipteric†. Plate XVIII, the *norma facialis*, indicates how much of the frontal bulge is really due to glabellar protrusion, and how little nasal resemblance the Moriori has to the Negro.

We conclude this series of photographs with reproductions of two crania of some interest. Plate XIX, No. 765^{17}, shows a cranium with marked para-occipital processes articulating with the atlas. Macdonell has figured in Plate X of his memoir on the Liverpool Street English crania‡ a case of unilateral para-occipital process of a like character, the present Moriori instance is bilateral.

Plates XX, XXI and XXII of a male skull, No. 765^{2}, illustrate how without surgical aid recovery from a serious injury is possible in a primitive people. The

* The pear-shaped cranium of Plate XI should be compared with the pear-shaped cranium Plate XI of Macdonell's paper, *Biometrika*, Vol. v. p. 104. The reader will understand at once what we mean by the heavy temporal lines and the depression of the *facies temporalis* in the Moriori. Indeed, if the reader will compare the photographs of the *norma verticalis* in Macdonell's two papers (*Biometrika*, Vol. III. p. 216, and Vol. v. p. 104) on the English skull, with our Plate XI, he will appreciate why the Moriori horizontal type contour differs so essentially from the English horizontal type contour. I regret that I have not provided more Moriori photographs illustrating this essential difference. In the Moriori crania the *norma verticalis* shows the temporal ridges, in the English they rarely appear at all.

† The curved dental arch is also worthy of note.

‡ *Biometrika*, Vol. v. p, 104.

left ramus of the mandible has been broken right across about the level of the alveolar ridge, possibly sheared across by a chin or side blow. Plate XX shows the right profile with the ramus and right side of the corpus normal; the only very obvious defect being the shortening of the mandible relative to the upper jaw. Plate XXI, the *norma facialis*, shows the asymmetry produced by the accident, the whole of the lower jaw is tilted. The source of this is explained on examining Plate XXII with the left profile; we see that the ramus has been broken right across, and both corpus and ramus on this side have ceased to develop properly. The posterior edge of the ramus is no longer nearly vertical as on the right profile, but slopes forward to a diminutive perigonal region. The whole mandible has, so to speak, been rotated round a vertical axis to allow for the dwarfing of the left side of the corpus, thus causing the upper jaw to protrude as seen in the profiles, and further has been rotated round a medial horizontal axis to allow for the dwarfing of the upper portion of the left ramus; the combination of these two rotations has given the remarkable skew effect produced in the *norma facialis*. The subject must have had a difficult time during life, but appears to have long survived his original trouble.

10. *Conclusions.* While I am by no means fully satisfied with this study of the Moriori skull and see many ways in which, if I could restart this investigation, it might be substantially improved, yet I must plead that what I have done has been sufficiently laborious to occupy fully my year's tenure of the Crewdson Benington Studentship. I can only say that experience has taught me certain difficulties which could be rectified should opportunity again offer, and that the detailed account I have given of these difficulties may aid others who plan cranial contour work. Further I hope to have provided an account of the Moriori skull, based on wider data than have hitherto been available; it will, I trust, in the future assist in settling racial affinities as more and more cranial material is reported on in a standardised form. If the racial affinities I find in the Moriori do not wholly coincide with the views of more authoritative anthropologists, they may at least serve as suggestions for the further examination of the primitive races who still or till recently bordered the Pacific Ocean. I feel confident that in the craniology of these races will be found a definite key to the evolution of man in prehistoric times; there have, I believe, been more extensive folk-wanderings, possibly rendered feasible by geological changes, than we have yet recognised. Craniological material from the outlying islands, and the fringes of the Pacific continents—if feasible of extinct races—is what one must above all things desire.

MISCELLANEA.

Note on the Probable Error of the Coefficient of Correlation in the Variate Difference Correlation Method.

By A. RITCHIE-SCOTT, B.Sc.

In *Biometrika*, Vol. x. p. 278, Dr Anderson of Petrograd has given the following expression for the probable error of the correlation coefficient between differences of the mth order.

Probable error of ${}_m r_{xy} = \cdot 67449\, {}_m\sigma_{r_{xy}}$

$$= \cdot 67449 \frac{1 - {}_m r^2_{xy}}{\sqrt{n-m}} \left\{ \frac{1}{n-m} \left[n - m + 2(n-m-1)\left(\frac{m}{m+1}\right)^2 \right.\right.$$

$$\left.\left. + 2(n-m-2)\left(\frac{m \,.\, m-1}{m+1\,.\,m+2}\right)^2 + 2(n-m-3)\left(\frac{m \,.\, m-1\,.\,m-2}{m+1\,.\,m+2\,.\,m+3}\right)^2 + \text{etc.} \right] \right\},$$

where m = order of difference,

n = size of original population (i.e. number of years in series of observations etc.).

In this formula the population is supposed to be reduced by unity at each difference taken. It is, however, found more convenient in practice to keep the population constant by taking in an additional observation at each successive operation of taking the difference. If we denote this constant population by n' the expression within the brackets reduces to

$$\frac{1}{n'}\left[n' + 2(n'-1)\left(\frac{m}{m+1}\right)^2 + 2(n'-2)\left(\frac{m \,.\, m-1}{m+1\,.\,m+2}\right)^2 + 2(n'-3)\left(\frac{m \,.\, m-1\,.\,m-2}{m+1\,.\,m+2\,.\,m+3}\right)^2 + \text{etc.} \right]$$

$$= 1 + 2\left(1 - \frac{1}{n'}\right)\left(\frac{m}{m+1}\right)^2 + 2\left(1 - \frac{2}{n'}\right)\left(\frac{m \,.\, m-1}{m+1\,.\,m+2}\right)^2$$

$$+ 2\left(1 - \frac{3}{n'}\right)\left(\frac{m \,.\, m-1\,.\,m-2}{m+1\,.\,m+2\,.\,m+3}\right)^2 + \text{etc.}$$

$$= 2\left\{1 + \left(\frac{m}{m+1}\right)^2 + \left(\frac{m \,.\, m-1}{m+1\,.\,m+2}\right)^2 + \left(\frac{m \,.\, m-1\,.\,m-2}{m+1\,.\,m+2\,.\,m+3}\right)^2 + \text{etc.}\right\}$$

$$- 1 - \frac{2}{n'}\left\{\left(\frac{m}{m+1}\right)^2 + 2\left(\frac{m \,.\, m-1}{m+1\,.\,m+2}\right)^2 + 3\left(\frac{m \,.\, m-1\,.\,m-2}{m+1\,.\,m+2\,.\,m+3}\right)^2 + \text{etc.}\right\}$$

$$= 2\phi(m) - 1 - \frac{2f(m)}{n'}.$$

The series $\phi(m)$ and $f(m)$ have been summed as follows by Professor Pearson:

$$\phi(m) = 1 + \left(\frac{m}{m+1}\right)^2 + \left(\frac{m \,.\, m-1}{m+1\,.\,m+2}\right)^2 + \left(\frac{m \,.\, m-1\,.\,m-2}{m+1\,.\,m+2\,.\,m+3}\right)^2 + \text{etc.}$$

∴ Multiply both sides by $\left(\frac{\lfloor 2m}{\lfloor m \lfloor m}\right)^2$

$$\left(\frac{\lfloor 2m}{\lfloor m \lfloor m}\right)^2 \phi(m) = \left(\frac{\lfloor 2m}{\lfloor m \lfloor m}\right)^2 + \left(\frac{\lfloor 2m}{\lfloor m-1 \,.\, \lfloor m+1}\right)^2 + \left(\frac{\lfloor 2m}{\lfloor m-2 \,.\, \lfloor m+2}\right)^2 + \text{etc.}$$

$$= (C_m^{2m})^2 + (C_{m+1}^{2m})^2 + (C_{m+2}^{2m})^2 + \ldots (C_{2m}^{2m})^2$$

$$= \tfrac{1}{2}\sum_0^{2m} (C_r^{2m})^2 + \tfrac{1}{2}(C_m^{2m})^2$$

(since the number of terms is odd)

$$= \tfrac{1}{2}\{C_{2m}^{4m} + (C_m^{2m})^2\}$$

$$= \frac{1}{2}\left\{\frac{\lfloor 4m}{\lfloor 2m \,.\, \lfloor 2m} + \left(\frac{\lfloor 2m}{\lfloor m \lfloor m}\right)^2\right\},$$

$$\therefore\ \phi(m) = \frac{1}{2}\left\{\frac{\lfloor 4m}{\lfloor 2m \lfloor 2m} \times \left(\frac{\lfloor m \lfloor m}{\lfloor 2m}\right)^2 + 1\right\}$$

$$= \frac{1}{2}\left\{\lfloor 4m \left(\frac{\lfloor m}{\lfloor 2m}\right)^4 + 1\right\}.$$

The series

$$f(m) = \left(\frac{m}{m+1}\right)^2 + 2\left(\frac{m \,.\, m-1}{m+1 \,.\, m+2}\right)^2 + 3\left(\frac{m \,.\, m-1 \,.\, m-2}{m+1 \,.\, m+2 \,.\, m+3}\right)^2 + \text{etc.}$$

has the value $\frac{1}{4}m$, a discovery I made when starting to table it. It is a particular case of a more general series which may be summed as follows:

Consider the following identity:

$$\frac{(m + \overline{m-1})^2}{4m} = (m-1) + m \,.\, \left(\frac{m - \overline{m-1}}{m+m}\right)^2.$$

Multiply both sides by $\left(\frac{m - \overline{m-2}}{m + \overline{m-1}}\right)^2$.

Then

$$\frac{(m - \overline{m-2})^2}{4m} = (m-1)\left(\frac{m - \overline{m-2}}{m + \overline{m-1}}\right)^2 + m\left(\frac{m - \overline{m-1}}{m+m} \,.\, \frac{m - \overline{m-2}}{m + \overline{m-1}}\right)^2.$$

The process may be continued, thus

$$\frac{(m + \overline{m-2})^2}{4m} = (m-2) + m\left(\frac{m - \overline{m-2}}{m+m}\right)^2 = (m-2) + \frac{(m - \overline{m-2})^2}{4m}$$

$$= (m-2) + (m-1)\left(\frac{m - \overline{m-2}}{m + \overline{m-1}}\right)^2 + m\left(\frac{m - \overline{m-1}}{m+m} \,.\, \frac{m - \overline{m-2}}{m + \overline{m-1}}\right)^2.$$

Multiply both sides by $\left(\frac{m - \overline{m-3}}{m + \overline{m-2}}\right)^2$.

Then

$$\frac{(m - \overline{m-3})^2}{4m} = (m-2)\left(\frac{m - \overline{m-3}}{m + \overline{m-2}}\right)^2 + (m-1)\left(\frac{m - \overline{m-2}}{m + \overline{m-1}} \,.\, \frac{m - \overline{m-3}}{m + \overline{m-2}}\right)^2$$

$$+ m \,.\, \left(\frac{m - \overline{m-1}}{m+m} \,.\, \frac{m - \overline{m-2}}{m + \overline{m-1}} \,.\, \frac{m - \overline{m-3}}{m + \overline{m-2}}\right)^2.$$

Assume now that this operation has been performed q times and that as a result the following equation is identically true:

$$\frac{\{m-(m-\overline{q+1})\}^2}{4m} = (m-q)\left\{\frac{m-(m-\overline{q+1})}{m+(m-q)}\right\}^2 + (m-\overline{q-1})\left\{\frac{m-(m-\overline{q+1})}{m+(m-q)}\cdot\frac{m-(m-q)}{m+(m-\overline{q-1})}\right\}^2 + \text{etc.}$$

Operating as before, we have

$$\frac{\{m+(m-\overline{q+1})\}^2}{4m} = (m-\overline{q+1}) + \frac{\{m-(m-\overline{q+1})\}^2}{4m}$$

Multiplying by $\left\{\frac{m-(m-\overline{q+2})}{m+(m-\overline{q+1})}\right\}^2$ and inserting the value of $\frac{\{m-(m-\overline{q+1})\}^2}{4m}$ we have

$$\frac{\{m-(m-\overline{q+2})\}^2}{4m} = (m-\overline{q+1})\left\{\frac{m-(m-\overline{q+2})}{m+(m-\overline{q+1})}\right\}^2 + (m-q)\left\{\frac{m-(m-\overline{q+1})}{m+(m-q)}\cdot\frac{m-(m-\overline{q+2})}{m+(m-\overline{q+1})}\right\}^2 + (m-\overline{q-1})\left\{\frac{m-(m-\overline{q+1})}{m+(m-q)}\cdot\frac{m-(m-q)}{m+(m-\overline{q-1})}\cdot\frac{m-(m-\overline{q+2})}{m+(m-\overline{q+1})}\right\}^2 + \text{etc.}$$

Hence if the expansion be true for $q+1$ it will be true for $q+2$. But it has been shown true for 1 and 2 and it is therefore true generally.

If now in the expansion for $\frac{\{m-(m-\overline{q+1})\}^2}{4m}$ we put $q = m-1$, we ge

$$\frac{m}{4} = \left(\frac{m}{m+1}\right)^2 + 2\left(\frac{m}{m+1}\cdot\frac{m-1}{m+2}\right)^2 + 3\left(\frac{m}{m+1}\cdot\frac{m-1}{m+2}\cdot\frac{m-2}{m+3}\right)^2 + \ldots = f(m).$$

Collecting these results together we may now write the probable error of ${}_m r_{xy}$

$$= \cdot 67449\,\frac{1-{}_m r^2{}_{xy}}{\sqrt{n'}}\left\{2\phi(m) - 1 - \frac{m}{2n'}\right\}^{\frac{1}{2}}$$

Values of the function $(2\phi(m)-1)$ have been calculated for values of m from $m=1$ to $m=20$ and are given in the table below.

m	$2\phi(m)-1$	m	$2\phi(m)-1$
1	1·5000	11	4·2282
2	1·9444	12	4·4100
3	2·3100	13	4·5845
4	2·6265	14	4·7527
5	2·9094	15	4·9151
6	3·1673	16	5·0723
7	3·4059	17	5·2248
8	3·6289	18	5·3730
9	3·8390	19	5·5172
10	4·0383	20	5·6578

After $m = 20$, the approximate value

$$2\phi(m) - 1 = \sqrt{1\cdot 570796\,(m+\cdot 375)}$$

given by Stirling's Theorem is close enough for practical purposes, e.g. $m = 20$, it gives 5·6573 or 5·6578.

On certain Types of Compound Frequency Distributions in which the Components can be individually described by Binomial Series.

By KARL PEARSON, F.R.S.

Certain difficulties with regard to the interpretation of *negative* binomials, which are of constant occurrence in observational frequency series, have suggested the following investigation.

Consider a number u of binomial series of which the sth is $\nu_s(p_s + q_s)^n$ and let us suppose a frequency series compounded by adding together the rth terms of all these series, such will be the compound frequency series it is proposed to discuss. We can realise its nature a little more concretely by supposing n balls drawn out of a bag containing Np white and Nq black balls, N being very large as compared with n, or else each ball returned before a fresh draw, while the values of p and q change discontinuously at the $\nu_1 + 1$, $\nu_1 + \nu_2 + 1$, $\nu_1 + \nu_2 + \nu_3 + 1$, etc. draws. We shall take as origin the point at which the sum of the first terms of all the binomials may be supposed to be plotted. S will denote summation to u terms.

Let $N = S(\nu_s)$, and $N\mu_1'$, $N\mu_2'$ be the moments of the compound system about this origin, $N\mu_1$, $N\mu_2$ its moments about its mean. Thus

$$N\mu_1' = S(\nu_s n q_s),$$
$$N\mu_2' = S\{\nu_s(np_sq_s + n^2q_s^2)\}.$$

Hence
$$N\mu_2 = nS\{\nu_s q_s(1 - q_s)\} + n^2\left(S(\nu_s q_s^2) - \frac{\{S(\nu_s q_s)\}^2}{N}\right)$$
$$= N\mu_1' + \frac{n}{N}[nS\{\nu_s\nu_{s'}(q_s - q_{s'})^2\} - NS(\nu_s q_s^2)],$$

since $N = S(\nu_s)$. Accordingly, if σ be the standard deviation and m the mean of the compound series, i.e. $\mu_2 = \sigma^2$, $\mu_1' = m$, then

$$\frac{\sigma^2}{m} - 1 = \frac{1}{N}\left(\frac{nS\{\nu_s\nu_{s'}(q_s - q_{s'})^2\} - NS(\nu_s q_s^2)}{S(\nu_s q_s)}\right). \quad \text{.....(i)}$$

Now suppose we had endeavoured to fit a binomial $N(P + Q)^\kappa$ to the compound series, we should have had

$$m = \kappa Q, \qquad \sigma^2 = \kappa PQ,$$

and accordingly have found

$$Q = -\frac{1}{N}\left(\frac{nS\{\nu_s\nu_{s'}(q_s - q_{s'})^2\} - NS(\nu_s q_s^2)}{S(\nu_s q_s)}\right), \quad \text{.....(ii)}$$

$$\kappa = -n\frac{\{S(\nu_s q_s)\}^2}{nS\{\nu_s\nu_{s'}(q_s - q_{s'})^2\} - NS(\nu_s q_s^2)}. \quad \text{.....(iii)}$$

Thus, had we attempted to fit a binomial to the heterogeneous series, we should have found Q negative and P greater than unity provided

$$nS\{\nu_s\nu_{s'}(q_s - q_{s'})^2\} \text{ be } > NS(\nu_s q_s^2),$$

a condition which will frequently be found to be satisfied, especially if q_s be small and n large.

In the limit let us take $nq_s = m_s$ and q_s vanishingly small, i.e. suppose the sth binomial to be replaced by the Poisson series

$$e^{-m_s}\left(1 + m_s + \frac{m_s^2}{1.2} + \ldots + \frac{m_s^i}{\lfloor i} + \ldots\right),$$

then we have at once

$$\left.\begin{aligned} Q &= -\frac{S\{\nu_s\nu_{s'}(m_s - m_{s'})^2\}}{NS\{\nu_s m_s\}}, \\ P &= 1 + \frac{S\{\nu_s\nu_{s'}(m_s - m_{s'})^2\}}{NS(\nu_s m_s)}, \\ \kappa &= -\frac{(S\{\nu_s m_s\})^2}{S\{\nu_s\nu_{s'}(m_s - m_{s'})^2\}}. \end{aligned}\right\} \quad \text{.....(iv)}$$

Thus, if two or more Poisson's series be combined term by term *from the first*, then the compound will always be a negative binomial. This theorem was first pointed out to me by "Student" and suggested by him as a possible explanation of negative binomials occurring in

material which theoretically should obey the Law of Small Numbers, e.g. "Student's" own Haemacytometer counts*. Of course the negative binomial may quite conceivably arise from other sources† than heterogeneity, but if this be the source of its origin in the material of Bortkewitsch, Mortara and McKendrick‡, it is certainly most unfortunate that such material should have been selected to illustrate Poisson's limit to the binomial.

Now we know that the values about the mean of the successive moment coefficients of the binomial are

$$\left.\begin{aligned} \mu_2 &= npq, \\ \mu_3 &= npq\,(p-q), \\ \mu_4 &= npq\,(1+3\,(n-2)\,pq). \end{aligned}\right\} \quad \text{.....(v)}$$

Further the mean is at a distance nq from the first term p^n. We shall call this distance m.

Let μ_1', μ_2', μ_3' and μ_4' be the moment coefficients round the start of each binomial. Then

$$\left.\begin{aligned} \mu_1' &= nq, \qquad \mu_2' = npq + n^2q^2, \\ \mu_3' &= npq\,(p-q) + 3npq \times nq + n^3q^3, \\ \mu_4' &= npq\,(1+3\,(n-2)\,pq) + 4npq\,(p-q)\,nq + 6\,(npq)\,n^2q^2 + n^4q^4. \end{aligned}\right\} \quad \text{.....(vi)}$$

From these equations we deduce

$$\left.\begin{aligned} \mu_1' &= nq, \qquad \mu_2' - \mu_1' = n\,(n-1)\,q^2, \\ \mu_3' - 3\mu_2' + 2\mu_1' &= n\,(n-1)\,(n-2)\,q^3, \\ \mu_4' - 6\mu_3' + 11\mu_2' - 6\mu_1' &= n\,(n-1)\,(n-2)\,(n-3)\,q^4. \end{aligned}\right\} \quad \text{.....(vii)}$$

Now let

$a_1 = \mu_1'$ for the combined series,

$a_2 = \mu_2' - \mu_1'$ for the combined series,

$a_3 = \mu_3' - 3\mu_2' - 2\mu_1'$ for the combined series,

$a_4 = \mu_4' - 6\mu_3' + 11\mu_2' - 6\mu_1'$ for the combined series.

Then we have, if $\lambda_1 = \nu_1/N$, $\lambda_2 = \nu_2/N$:

$$\left.\begin{aligned} 1 &= \lambda_1 + \lambda_2, \\ \frac{a_1}{n} &= \lambda_1 q_1 + \lambda_2 q_2, \\ \frac{a_2}{n\,(n-1)} &= \lambda_1 q_1^2 + \lambda_2 q_2^2, \\ \frac{a_3}{n\,(n-1)\,(n-2)} &= \lambda_1 q_1^3 + \lambda_2 q_2^3, \\ \frac{a_4}{n\,(n-1)\,(n-2)\,(n-3)} &= \lambda_1 q_1^4 + \lambda_2 q_2^4. \end{aligned}\right\} \quad \text{.....(viii)}$$

Multiply each equation by q_1 and subtract from that below it and we find:

$$\left.\begin{aligned} \frac{a_1}{n} - q_1 &= \lambda_2\,(q_2 - q_1), \\ \frac{a_2}{n\,(n-1)} - \frac{a_1 q_1}{n} &= \lambda_2 q_2\,(q_2 - q_1), \\ \frac{a_3}{n\,(n-1)\,(n-2)} - \frac{a_2 q_1}{n\,(n-1)} &= \lambda_2 q_2^2\,(q_2 - q_1), \\ \frac{a_4}{n\,(n-1)\,(n-2)\,(n-3)} - \frac{a_3 q_1}{n\,(n-1)(n-2)} &= \lambda_2 q_2^3\,(q_2 - q_1). \end{aligned}\right\} \quad \text{.....(ix)}$$

* *Biometrika*, Vol. v. p. 356, and see also L. Whitaker's examination of these data, Vol. x. p. 48.

† Pearson, *Biometrika*, Vol. iv. p. 209.

‡ For an examination of Bortkewitsch and Mortara's instances see L. Whitaker, *loc. cit.* pp. 49–66. McKendrick has recently reached Poisson's series (*Proceedings of the London Mathematical Society*, Vol. xiii. (1914), pp. 405 *et seq*) without apparently recognising that he was on familiar ground, and has suggested its application to the frequencies obtained by counts of the bacilli ingested by leucocytes. He has fitted his series not by moments, but from the first two terms, and has failed to recognise that a large proportion of such leucocyte counts give also *negative* binomials.

Hence dividing each equation by the preceding one:

$$\frac{\dfrac{a_2}{n(n-1)} - \dfrac{a_1 q_1}{n}}{\dfrac{a_1}{n} - q_1} = \frac{\dfrac{a_3}{n(n-1)(n-2)} - \dfrac{a_2 q_1}{n(n-1)}}{\dfrac{a_2}{n(n-1)} - \dfrac{a_1 q_1}{n}} = \frac{\dfrac{a_4}{n(n-1)(n-2)(n-3)} - \dfrac{a_3 q_1}{n(n-1)(n-2)}}{\dfrac{a_3}{n(n-1)(n-2)} - \dfrac{a_2 q_1}{n(n-1)}} = q_2. \quad \text{.....(x)}$$

Writing $q_1 q_2 = P_2$ and $q_1 + q_2 = P_1$, we obtain

$$\left.\begin{aligned} a_2 - (n-1)a_1 P_1 + n(n-1)P_2 &= 0, \\ a_3 - (n-2)a_2 P_1 + (n-1)(n-2)a_1 P_2 &= 0, \\ a_4 - (n-3)a_3 P_1 + (n-2)(n-3)a_2 P_2 &= 0, \end{aligned}\right\} \quad \text{.....(xi)}$$

three equations to determine n, P_1 and P_2.

Eliminating $-P_1$ and P_2 we find:

$$\begin{vmatrix} a_2 & (n-1)a_1 & n(n-1) \\ a_3 & (n-2)a_2 & (n-1)(n-2)a_1 \\ a_4 & (n-3)a_3 & (n-2)(n-3)a_2 \end{vmatrix} = 0, \quad \text{.....(xii)}$$

which expanded gives us the cubic for n:

$$n^3(2a_1a_2a_3 - a_2^3 - a_1^2a_4 + a_2a_4 - a_3^2) + n^2(-12a_1a_2a_3 + 7a_3^3 + 4a_1^2a_4 - 3a_2a_4 + 4a_2^2) + n(22a_1a_2a_3 - 16a_2^3 - 5a_1^2a_4 + 2a_2a_4 - 3a_3^2) + (-12a_1a_2a_3 + 12a_2^3 + 2a_1^2a_4) = 0. \quad \text{...(xii)}^{bis}$$

A root of this cubic substituted in the first two equations of (xi) will give P_1 and P_2 and then the quadratic

$$p^2 - P_1 p + P_2 = 0 \quad \text{.....(xiii)}$$

will determine the two values q_1 and q_2 corresponding to the value of n. The first two equations of (viii) then complete the solution by providing λ_1 and λ_2.

Until the roots of the cubic (xii)^{bis} have been discussed we can only assume that three solutions are possible. As a matter of fact in the examples so far dealt with some of these solutions have usually to be discarded.

For the special case of Poisson's limit to the binomial, we make n indefinitely large, q indefinitely small, and $nq = m$ finite. Hence equations (viii) become

$$\left.\begin{aligned} 1 &= \lambda_1 + \lambda_2, \\ a_1 &= \lambda_1 m_1 + \lambda_2 m_2, \\ a_2 &= \lambda_1 m_1^2 + \lambda_2 m_2^2, \\ a_3 &= \lambda_1 m_1^3 + \lambda_2 m_2^3, \\ a_4 &= \lambda_1 m_1^4 + \lambda_2 m_2^4, \end{aligned}\right\} \quad \text{.....(xiv)}$$

leading to

$$\begin{aligned} a_2 - a_1 Q_1 + Q_2 &= 0, \\ a_3 - a_2 Q_1 + a_1 Q_2 &= 0, \\ a_4 - a_3 Q_1 + a_2 Q_2 &= 0, \end{aligned}$$

if $$Q_1 = m_1 + m_2, \qquad Q_2 = m_1 m_2.$$

Thus we find:

$$Q_1 = (a_3 - a_1 a_2)/(a_2 - a_1^2),$$
$$Q_2 = (a_3 a_1 - a_2^2)/(a_2 - a_1^2),$$

subject to the condition*

$$a_4(a_2 - a_1^2) + 2a_1 a_2 a_3 - a_3^3 - a_2^3 = 0. \quad \text{.....(xv)}$$

Hence m_1 and m_2 are roots of

$$m^2(a_2 - a_1^2) - m(a_3 - a_1 a_2) + a_3 a_1 - a_2^2 = 0. \quad \text{.....(xvi)}$$

* Of course equations of condition hold for the 5th and higher moments in the case of the two binomial components. But they are of small service as the probable errors of these high moments are usually very considerable.

Further $$\lambda_1 = \frac{\mu_1' - m_2}{m_1 - m_2}, \quad \lambda_2 = \frac{\mu_1' - m_1}{m_2 - m_1}, \qquad \text{(xvii)}$$

which determine λ_1 and λ_2.

It is thus quite easy to resolve a series into the sum of two Poisson's binomial limits provided the roots of the above quadratic are real. As illustration I take "Student's" first count of yeast cells on the 400 squares of a haemacytometer. He found:

No. of yeast cells	0	1	2	3	4	5	Total
Frequency	213	128	37	18	3	1	400

giving: mean $= \mu_1' = \cdot6825$, $\mu_2 = \cdot8117$, $\mu_3 = 1\cdot0876$.

From the above values of "Student" I determined

$$\mu_2' = 1\cdot2775, \qquad \mu_3' = 3\cdot0675.$$

Whence $$a_1 = \cdot6825, \qquad a_2 = \cdot5950, \qquad a_3 = \cdot6000,$$

and the resulting quadratic is

$$\cdot129{,}194m^2 - \cdot193{,}913m + \cdot055{,}475 = 0.$$

The roots are $m_1 = \cdot3847$ and $m_2 = 1\cdot1163$, leading to $\lambda_1 = \cdot59{,}295$ and $\lambda_2 = \cdot40{,}705$, or, the series has for its two components

$$\nu_1 = 237\cdot18, \qquad m_1 = \cdot3847,$$
$$\nu_2 = 162\cdot82, \qquad m_2 = 1\cdot1163.$$

Calculating out the Poisson's series for these components we have:

No. of yeast cells	0	1	2	3	4	5	6	7
1st Compt.	161·44	62·11	11·95	1·53	·15	·01	·00	·00
2nd Compt.	53·32	59·52	32·22	12·36	3·45	·77	·14	·02
Round totals	215	122	44	14	4	1		
Observed	213	128	37	18	3	1		

The test for "goodness of fit" for these six groups gives $\chi^2 = 2\cdot82$ and $P = \cdot73$, or the fit is very good. The negative binomial gave $P = \cdot52$ and a single Poisson's series only $P = \cdot04$. But the double Poisson's series places of course one constant more at our disposal than the binomial, and we can do still better with a double binomial, as we have four constants and only six frequencies, while the double Poisson has three constants to six frequencies. It is clear that neither of the above components forming 41 % and 59 % of the total number of cells, and having their means at ·3847 and 1·1163 instead of ·6825, gives any idea of a dominant constitution in the solution sampled. If in this case heterogeneity accounts for the negative binomial, then the difference of the components is not slight, and the heterogeneity being gross would indicate some considerable failure in technique.

If we assume that the counts with a haemacytometer ought to follow the Poisson distribution, —and this seems to be theoretically probable,—then the criterion of the binomial might well be adopted to ascertain the possibility of some failure in technique. The actual binomial in "Student's" first case should be $(\frac{1}{100} + \frac{399}{400})^{273}$ and any binomial with p very small and $np = \cdot6825$ would effectively represent the series; we could not anticipate getting $n = 273$ and $p = \frac{1}{100}$ closely from the data, but we might certainly anticipate a *positive* binomial, if the theory of a Poisson distribution be correct. If on the other hand we say in this and many similar cases that the negative binomial arises from heterogeneity, then it appears to me that we have saved our theory at the expense of our technique. I propose now to test this point further by considering the component binomials. If the theory of heterogeneity be correct, unless it be very

manifold, we might anticipate two binomial components, $\nu_1(p_1+q_1)^n + \nu_2(p_2+q_2)^n$, with n positive and large, both q_1 and q_2 being small, while n_1q_1 and n_2q_2 would be approximately ·3847 and 1·1163, the frequencies ν_1 and ν_2 being roughly in the ratio of 3 to 2.

Returning to "Student's" data we find $\mu_4' = 8{\cdot}9275$, whence

$$a_1 = {\cdot}6825, \qquad a_2 = {\cdot}5950, \qquad a_3 = {\cdot}6000, \qquad a_4 = {\cdot}4800.$$

Substituting in (xii)^{bis} we obtain for the cubic:

$${\cdot}021{,}3269n^3 + {\cdot}028{,}2321n^2 + {\cdot}363{,}3020n + {\cdot}051{,}0825 = 0.$$

This has three real roots, approximately.

$$n' = 4{\cdot}89997, \quad n'' = -{\cdot}14234, \quad \text{and} \quad n''' = -3{\cdot}43390.$$

We will consider in succession these cases: (i) $n' = 4{\cdot}89997$. The first two equations of (xi) provide

$${\cdot}5950 - 2{\cdot}66173P_1 + 19{\cdot}10974P_2 = 0,$$
$${\cdot}6000 - 1{\cdot}72548P_1 + 7{\cdot}71894P_2 = 0,$$

leading to $P_1 = {\cdot}55304$, $P_2 = {\cdot}04590$, and the quadratic

$$q^2 - {\cdot}55304q + {\cdot}04590 = 0,$$

whence we deduce the binomial factors

$$q_1 = {\cdot}4514, \quad p_1 = {\cdot}5486, \quad \text{and} \quad q_2 = {\cdot}1017, \quad p_2 = {\cdot}8983.$$

The first two equations of (viii) give

$$1 = \lambda_1 + \lambda_2, \qquad {\cdot}6825/4{\cdot}89997 = {\cdot}451{,}355\lambda_1 + {\cdot}101{,}685\lambda_2,$$

leading to

$$\lambda_1 = {\cdot}107{,}535, \qquad \lambda_2 = {\cdot}892{,}465,$$

or, in a population of 400,

$$\nu_1 = 43{\cdot}014, \qquad \nu_2 = 356{\cdot}986.$$

Accordingly the compound series is given by

$$43{\cdot}014\,({\cdot}5486 + {\cdot}4514)^{4\cdot89997} + 356{\cdot}986\,({\cdot}8983 + {\cdot}1017)^{4\cdot89997},$$

with means of the components at

$$m_1 = 2{\cdot}2118 \quad \text{and} \quad m_2 = {\cdot}4983.$$

We see that neither the sizes of the component populations nor their means have any relation to the previously discussed component Poisson series; further the present series* diverge widely from Poisson series, n is not large nor q_1 or q_2 very small. Calculated to the nearest whole numbers we obtain:

No. of yeast cells	0	1	2	3	4	5
1st Compt.	2	9	15	12	4	1
2nd Compt.	211	117	26	3	0	0
Combination	213	126	41	15	4	1
Observed	213	128	37	18	3	1

which leads to $\chi^2 = 1{\cdot}27$ and $P = {\cdot}93$.

Thus the fit is excellent, but it does not correspond to the heterogeneity of a double Poisson series.

(ii) $n'' = -{\cdot}14234$. Here the first two equations of (xi) provide

$${\cdot}5950 + {\cdot}77965P_1 + {\cdot}16260P_2 = 0,$$
$${\cdot}6000 + 1{\cdot}27469P_1 + 1{\cdot}75727P_2 = 0,$$

and give $P_1 = -{\cdot}81529, \quad P_2 = {\cdot}24996,$

with $q^2 + {\cdot}81529q + {\cdot}24996 = 0.$

* It should be noted that such fractional binomial series tend ultimately to become negative, although with negligibly small frequencies.

This gives imaginary values of q_1 and q_2 and thus the solution can for the present purpose be discarded.

(iii) $n''' = -3{\cdot}43390$. We deduce

$$\cdot5950 + 3{\cdot}02614P_1 + 15{\cdot}22557P_2 = 0,$$

$$\cdot6000 + 3{\cdot}23317P_1 + 16{\cdot}44372P_2 = 0,$$

giving $P_1 = -1{\cdot}21441, \quad P_2 = \cdot20229$

and $q^2 + 1{\cdot}21441q + \cdot20229 = 0.$

Hence $q_1 = -1{\cdot}0151, \quad p_1 = 2{\cdot}0151, \quad q_2 = -\cdot1993, \quad p_2 = 1{\cdot}1993.$

The λ equations are

$$1 = \lambda_1 + \lambda_2, \qquad -\cdot6845/3{\cdot}4339 = -1{\cdot}0151\lambda_1 - \cdot1993\lambda_2,$$

leading to $\lambda_1 = \cdot000043, \quad \lambda_2 = \cdot999957,$

or, $\nu_1 = \cdot0172, \quad \nu_2 = 399{\cdot}9828.$

Thus the component series is

$$\cdot0172\,(2{\cdot}0151 - 1{\cdot}0151)^{-3{\cdot}4339} + 399{\cdot}9828\,(1{\cdot}1993 - \cdot1993)^{3{\cdot}4339},$$

with means at $m_1 = 3{\cdot}4858$ and $m_2 = \cdot6847.$

The first of these components is negligible, it contains roughly only ·02 individuals in 400, and the second is sensibly identical with the negative binomial obtained by "Student," i.e.

$$400\,(1{\cdot}1893 - \cdot1893)^{-3{\cdot}8054},$$

with slightly modified constants. It provides:

No. of yeast cells	0	1	2	3	4	5
Calculated	214	122	45	14	4	1
Observed	213	128	37	18	3	1

leading to $\chi^2 = 3{\cdot}12$ and $P = \cdot68$, which for all practical purposes is as good as the double Poisson.

Conclusions. It having been suggested that the appearance of *negative* binomials as better "fits" than Poisson's series for material that is supposed to follow the law of small numbers is due to heterogeneity, formulae have been provided for testing whether this heterogeneity is due to a second component. If so this component should be small and the first component should substantially agree with the primary Poisson's series. The smallness of the second component would measure the goodness of the technique in haemacytometer or opsonic index counts. Applied to "Student's" first series of counts of yeast cells we obtain (*a*) two Poisson's series neither of which dominates the data or approximates to the primary Poisson's series; (*b*) two positive binomials, neither of which has any approach to a Poisson series or any agreement with the components of (*a*); and lastly (*c*) two *negative* binomials, one of which dominates the series, and agrees with the primary negative binomial: This investigation as far as it goes suggests either that "Student's" first count is really described *homogeneously* by a negative binomial, or, if it be heterogeneous, then the heterogeneity is manifold, and no weight can be given to the results of fitting by the primary Poisson's series.

The general numerical discussion by the formulae of this paper of a variety of data assumed to follow the "law of small numbers" is in hand and will shortly be published.